Recent contributions to Mechanism Design: A Highly Selective Review

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Mechanism Design

- part of game theory devoted to "reverse engineering"
- usually we take game as given
  - try to predict the outcomes it generates in equilibrium
- in MD, we (the "planner") start with outcome(s) *a* we want as a function of underlying state of the θ (social choice correspondence $f : \Theta \to\to \text{A}$)
  - difficulty: we may not know state
  - try to design a game (mechanism) whose equilibrium outcomes same as those prescribed by social choice function

    mechanism *implements* SCC

- Goes back (at least) to 19^{th} century Utopians
  - can one design “humane” alternative to laissez-faire capitalism?
- Socialist Planning Controversy 1920s-40s
  - can one construct a centralized planning mechanism that replicates or improves on competitive markets?
    - O. Lange and A. Lerner: yes
    - L. von Mises and F. von Hayek: no
  - brought to fore 2 major themes
    - incentives
    - information

Modern mechanism-design theory dates from 2 papers in early 1960's

- L. Hurwicz (1960)
  - introduced basic concepts
    - mechanism
    - informational decentralization
    - informational efficiency
- W. Vickrey (1961)
  - exhibited a particular but important mechanism: 2^{nd} price auction

Since then, field has expanded dramatically

- vast literature, ranging from
  - very general
    - possible outcomes ↔ abstract set of social alternatives
      - (at least 10 major survey articles and books in last dozen years or so)
  - quite particular
    - design of bilateral contracts between buyer and seller
    - (several recent books on contract theory, including Bolton-Dewatripont (2005) and Laffont-Martimort (2002))
    - design of auctions for allocating a good among competing bidders
      - (several recent books - - Krishna (2002), Milgrom (2004), Klemperer (2004))
  - far too much recent work to survey properly here
  - will pick 3 specific developments (both general and particular)

- interdependent values in auction design
- robustness of mechanisms
- indescribable states, renegotiation and incomplete contracts

Interdependent values in auction design

- seller has 1 good
- *n* potential buyers
- how to allocate good *efficiently*?

  (to buyer who values good the most)

  i.e., how to implement SCC that selects efficient allocations

In *private* values case (each buyer's valuation is independent of others' information),

Vickrey (1961) answered question:

- 2nd price auction is efficient
  - buyers submit bids
  - winner is high bidder
  - winner pays 2nd highest bid
- if v_i is buyer i's valuation, optimal for him to bid

$$b_i = v_i$$

- winner will have highest valuation

What if values are *interdependent*?

- each buyer *i* gets private signal s_i (one-dimensional)
- buyer *i*'s valuation is $v_i\left(s_i, s_{-i}\right)$
- buyer *i* no longer knows own valuation
  - so can't bid valuation in equilibrium
  - might bid *expected* valuation, but this not enough for efficiency: might have

$$E_{s_{-i}} v_i\left(s_i, s_{-i}\right) > E_{s_{-j}} v_j\left(s_j, s_{-j}\right)$$

but

$$v_i\left(s_i, s_{-i}\right) < v_j\left(s_j, s_{-j}\right)$$

- consider auction in which
  - each buyer *i* announces $\hat{s}_i$
  - winner is buyer *i* for which
$$v_i\left(\hat{s}_i, \hat{s}_{-i}\right) > \max_{j \neq i} v_j\left(\hat{s}_i, \hat{s}_{-i}\right)$$
  - winner pays
$$v_i\left(s_i^*, \hat{s}_{-i}\right) = \max_{j \neq i} v_j\left(s_i^*, \hat{s}_{-i}\right).$$
- if
$$\frac{\partial v_i}{\partial s_i} > \frac{\partial v_j}{\partial s_i} \text{ whenever } v_i = v_j$$
then equilibrium to bid $\hat{s}_i = s_i$, so auction efficient
- difficulty: requires auction designer to know signal spaces and functional forms $v_i\left(s_i, s_{-i}\right)$

- Instead, consider auction in which
  - each buyer *i* makes contingent bid

    $b_i\left(v_{-i}\right) =$ *i*'s bid if other buyers' valuations revealed to be v_{-i}
  - calculate fixed point $\left(v_1^\circ, \ldots, v_n^\circ\right)$ such that

    $$v_i^\circ = b_i\left(v_{-i}^\circ\right)$$
  - winner is buyer *i* such that

    $$v_i^\circ > \max_{j \neq i} v_j^\circ$$
  - winner pays

    $$b_i\left(v_{-i}^*\right) = \max_{j \neq i} v_j^*$$

    where $v_j^* = b_j\left(v_{-j}^*\right) \quad j \neq i$
- under basically same conditions as before, in equilibrium buyer *i* with signal s_i bids true contingent valuation

  $$b_i\left(v_{-i}\left(s_i, s_{-i}\right)\right) = v_i\left(s_i, s_{-i}\right) \text{ for all } s_{-i}$$
- auction efficient

Open Problem: How to handle multiple goods with complementarities in dynamic auction (dynamic auctions like English auction are easier on buyers than once-and-for-all auctions like 2nd –price auction)

Robust Mechanism Design

auction in which buyer *i* bids $b_i(v_{-i})$ is "robust" or "independent of detail" in sense that

- it doesn't matter whether auction designer knows buyers' signal spaces or functional forms $v_i(s_i, s_{-i})$
- it doesn't matter what buyer *i* believes about the distribution of s_{-i}
  - optimal for buyer *i* to set

$$b_i\left(v_{-i}\left(s_i, s_{-i}\right)\right) = v_{-i}\left(s_i, s_{-i}\right) \text{ for all } s_{-i}$$

regardless of *i*'s belief about s_{-i}

  - i.e., bidding truthfully is an *ex post equilibrium*

(remains equilibrium even if *i knows* s_{-i})

Why is robustness important?

- common in Bayesian mechanism design to identify buyer *i*'s possible *types* with his possible *preferences (*common more generally than justification)

  set of possible types $\leftrightarrow$ set of possible preferences Θ

- but this has extreme implication: if you know *i*'s preferences, know his beliefs over other's types
  - no reason why this should hold
  - overly strong consequences:

    in auction model above, if signals correlated, auctioneer can attain efficiency and extract all buyer surplus without any conditions such as

    $$\frac{\partial v_i}{\partial s_i} > \frac{\partial v_j}{\partial s_i}$$

    (Crémer and McLean (1985))
  - As Neeman (2001) and Heifetz and Neeman (2004) shows, Crémer-McLean result goes away for suitably richer type spaces (preference corresponds to multiple possible beliefs)
- more generally, no reason why auction designer should know what buyers' type spaces are

Given SCC $f : \Theta \rightarrow\rightarrow A$, can we find mechanism for which, regardless of type space associated with preference space Θ, there always exists *f*-optimal equilibrium?

(robust partial implementation)

- sufficient condition: *f* partially implementable in *ex post* equilibrium, i.e., there exists mechanism that always has *f*-optimal *ex post* equilibrium (may be other equilibria)
  - *ex post* equilibrium reduces to dominant strategy equilibrium with private values
- Bergemann and Morris (2004) show that condition not necessary

But *ex post* partial implementability *is* necessary for robust partial implementation if

- outcome space takes form

$$X \times Y_1 \times \cdots \times Y_n$$

"common" outcome (public good) — X

"private transfers" — $Y_1 \times \cdots \times Y_n$

agent i cares just about (x, y_i)

- satisfied in above auction model (and, more generally, in quasilinear models)

- So far have been concentrating on *partial* implementation (not all equilibria have to be *f*-optimal)
- But unless planner sure that agents will play *f*-optimal equilibrium, more appropriate concept is *full* implementation: *all* equilibria of mechanism must be *f*-optimal

- key to full implementation is some species of *monotonicity*
  - full implementation in Nash equilibrium (agents have complete information) requires standard monotonicity:

social choice function (SCF) f monotonic if, for all $\alpha:\Theta\to\Theta$ and $\theta\in\Theta$

for which $f\left(\alpha\left(\theta\right)\right)\neq f\left(\theta\right),$ there exist i and $a\in A$

such that

and $u_i\left(a,\theta\right)>u_i\left(f\left(\alpha\left(\theta\right)\right),\theta\right)$

$$u_i\left(f\left(\alpha\left(\theta\right)\right),\alpha\left(\theta\right)\right)\geq u_i\left(a,\alpha\left(\theta\right)\right)$$

  - analogous condition for Bayesian implementation- -Postlewaite and Schmeidler (1986)

    (agents have incomplete information)

Bergemann and Morris (2005):

- identify *ex post monotonicity* as key to ex post full implementabilty

  f *ex post* monotonic if for all α such that $f \circ \alpha \neq f$, there exist $i, \theta,$ and a

  such that

  $$u_i(a,\theta) > u_i\left(f\left(\alpha(\theta)\right),\theta\right)$$

  and

  $$u_i\left(f\left(\theta_i',\alpha_{-i}(\theta_{-i})\right),\left(\theta_i',\alpha_{-i}(\theta_{-i})\right)\right) \geq u_i\left(a,\left(\theta_i',\alpha_{-i}(\theta_{-i})\right)\right) \text{for all } \theta_i'.$$

- show: in economic settings SCF f for $n \geq 3$ is *ex post* fully implementable if and only if it satisfies *ex post* monotonicity and *ex post* incentive compatibility

  $$u_i\left(f(\theta),\theta\right) \geq u_i\left(f\left(\theta_i',\theta_{-i}\right),\theta\right) \text{ for all } i, \theta_i', \theta.$$

- *ex post* equilibrium is refinement of Nash equilibrium but *ex post* monotonicity doesn't imply standard monotonicity (nor is it implied)
  - although *ex post* equilibrium is more demanding solution concept, makes ruling *out* equilibria easier
- Notable SCC where *ex post* monotonicity but not monotonicity satisfied: efficient allocation rule in interdependent values auction model when $n \geq 3$
  - generalization of 2nd-price auction fully *ex post* implements this rule
  - Berliun (2003) shows that hypothesis $n \geq 3$ is important: there exist inefficient *ex post* equilibria in case $n = 2$.

- But *ex post* full implementation not quite enough
  - rules out nonoptimal *ex post* equilibria
  - but there could be other sorts of nonoptimal equilibria
- really need *robust full implementation*:
- Can $f:\Theta \rightarrow\rightarrow A$ be implemented by mechanism such that, regardless of type space associated with Θ, all equilibria are *f*-optimal?
- Bergemann and Morris (2003) show that condition called *robust monotonicity* is what is needed to ensure robust full implementation
  - stronger than both *ex post* monotonicity and standard monotonicity
- From Stephen Morris seminar, believe that for $n \geq 3$, generalized 2nd price auction robustly fully implements the efficient SCC as long as not "too much" interdependence

- so far, “robustness” requirement pertains to *mechanism designer*
  - may not know agents’ type spaces
- also recent contributions in which robustness pertains to *agents playing mechanism*

- large literature considering implementation in various *refinements* of Nash equilibrium
  - allows implementation of SCCs that are not monotonic in standard sense
- any species of Nash equilibrium entails that agents have common knowledge of one another's preferences
- but what if agents are (slightly) uncertain about state of world?
  - which SCCs are robust to this uncertainty?
- Chung and Ely (2003) show that only *monotonic* SCCs can be robustly implemented in this sense

Example (Jackson and Srivastava (1996))

$$n = 2 \qquad A = \{a, b, c\} \qquad \Theta = \{\theta, \theta'\}$$

θ

1	2
c	a
a	b
b	c

θ'

1	2
c	a
a	c
b	b

$f(\theta) = a$

$f(\theta') = c$

not monotonic

f implemented in undominated Nash equilibrium by

	m_2	m_2'
m_1	a	a
m_1'	b	c

(m_1, m_2) unique equil in θ

(m_1', m_2') unique equil in θ'

- but m_2 is dominated in state θ' only if player 2 *sure* that state is θ'
- if small probability that state is θ, mechanism no longer implements *f*
- in fact, *no* mechanism can implement *f* because nonmonotonic

Open problem: Implications of robustness for applications

Indescribable States, Renegotiation and Incomplete Contracts

- incomplete contracts literature studies how assigning *ownership* (or control) of productive assets affects *efficiency* of outcome
- For efficiency to be in doubt, must be some constraint on contracting

  (i.e., on mechanism design)

- In this literature, constraint is *incompleteness* of contract
  - contract not as fully contingent on state of world as parties would like
- Reason for incompleteness
  - parties plan to trade a good in future
  - do not know characteristics of good (state) at the time of contracting (although common knowledge at time of trade)
  - contract cannot even describe set of possible states (too vast)
  - so contract cannot be contingent

Nevertheless, we have:

Irrelevance Theorem (Maskin and Tirole (1999)):

If parties are risk averse and can assign probability distribution to their future *payoffs*, then can achieve same expected payoffs as with fully contingent contract (even though cannot describe possible states in advance)

Idea:

- design contracts that specify *payoff* contingencies
- later, when state of world realized, can fill in *physical* details
- possible problem: incentive compatibility
  - will it be in parties interest to specify physical details truthfully?
  - but if

$$\text{different states} \leftrightarrow \text{different preferences,}$$

can design mechanism to ensure incentive compatibility

Where does risk aversion come in?

Answer: helps with incentive compatibility

- if parties are supposed to play (m_1, m_2) in θ but 1 plays m_1' instead, must be punished
- but if (m_1', m_2') is equilibrium play in θ', then not clear from (m_1', m_2) who has deviated
- resolution: punish them *both* with inefficient outcome *a*.

But what if parties can renegotiate *ex post* ?

- not an issue when designer is third party; here parties themselves design contract
- why settle for *a ?*
- will renegotiate *a* to something Pareto optimal
- renegotiation interferes with effective punishment
- in Segal (1999) and Hart and Moore 1999), renegotiation is so constraining that mechanisms are useless

Risk aversion

- Pareto frontier (in utility space) is strictly concave
- so if *randomize* between 2 Pareto optimal points, generate point in interior (bad outcome)
- so can punish both parties after all.

Open problem:

How to provide solid foundation for incomplete contracts?