The Promise and Problems of (Auction) Market Design

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Market design is a kind of economic engineering, utilizing laboratory research, game theory, algorithms, simulations, and more. Its challenges inspire us to rethink longstanding fundamentals of economic theory.

Two Areas of Market Design

- Matching Markets without Money
  - Doctors & Hospitals
  - School assignments
  - Kidneys
  - Course allocation
- Auction Markets: Matching and Pricing and More
  - Radio spectrum
  - Power (electricity and gas)
  - "Commodities"
  - Internet advertising

Revisiting Foundations

- How Should Products/Contracts Be Defined?
  - "A commodity is characterized by its physical properties, the date at which it will be available, and the location at which it will be available." (Debreu, 1959)
- When (and How) Should "Different" Markets Be Linked?
  - Always/never, as in General Equilibrium Theory?
- What Messages Should a Mechanism Use?
  - Revelation principle: "any equilibrium outcome of an arbitrary mechanism can be replicated by an incentive-compatible direct mechanism." (2007 Nobel citation)
- How Should Incentives Be Provided?
  - Use "an incentive-compatible direct mechanism"?

Product Definitions

Product Definitions in Practice

- Wheat
  - From *The Book of Wheat* by Peter Dondlinger, published 1908: "…for each transaction they would analyze a sample to determine its value. The measurement costs were very high."
- Diamonds
  - BHP Billiton auction: 19 "deals" are sold in "splits," with "book" adjustments. (Cramton, Dinkin & Wilson, 2009)
- Radio spectrum auctions
  - Bandwidth, geographic area and …
- Advertising impressions
  - Keywords, interests, demographics, behavioral history, etc.

Effects of Product Definition

- Wheat example. Setting standards…
  - Reduced measurement costs (and/or adverse selection)
  - Reduced shipping cost (grain cars on trains)
  - Enabled futures markets for wheat
- …but finer classifications may lead to…
  - Better matching of goods to buyers
  - More efficient quality choices by suppliers
  - Thinner markets within each classification
- Online advertising examples
  - Facebook: Cubs stadium merchandise
  - Yahoo/McDonald's "Happy Contract"
- Publishers' fears of "commoditization"

Product definition questions bleed into message design issues.

Message Spaces

Message Length Problem

- A direct mechanism requires reporting a value for every possible combination of licenses.
- In the US, FCC radio spectrum auctions may involve more than 1000 licenses.
  - Example – Auction 66: 1132 licenses
  - A report in such a mechanism conveys 2^{1132} numbers.
- Possible fixes?
  - Multi-round auctions.
  - Messages report only parameterized preferences.

Simplified Messages*

- Limited reporting changes the set of Nash equilibria.
  - Some equilibrium profiles may be eliminated, if the corresponding reports are *eliminated* by the simplification.
  - Some equilibrium profiles may be *added*, if all profitable deviations are eliminated by the simplification.
- A simplified mechanism avoids introducing new equilibria if it has the *outcome closure property*…

*Based on Milgrom (2009), "Simplified Mechanisms with an Application to Sponsored Search Auctions"

Outcome Closure Property (Formal)

Standard Set-up:

- Message profiles: $M = M_1 \times \ldots \times M_N$
- Outcome set is $X \subseteq X_1 \times \ldots X_N$.
- A *mechanism* is $\Omega = (M, \omega)$ with $\omega : M \rightarrow X$.
- Agent *j's* has utility payoff is $u_j : X_j \rightarrow \Re$.

New Definitions:

- Let *M'* be a subset of *M*. Then, $\Omega' = (M', \omega_{|M'})$ is a *simplification* of $\Omega = (M, \omega)$ and Ω is an *extension* of Ω'.
- A simplification has the *outcome closure property* if for every player *j* and every profile of restricted messages m_{-j} for players $-j$, $\text{cl}(\omega(M_j, m_{-j})) = \text{cl}(\omega(M'_j, m_{-j}))$.

Again, in Ordinary English

- A mechanism $\Omega=(M, \omega)$ is a pair consisting of a set of messages for each player and a function mapping messages to outcomes.
- A first mechanism is a *simplification* of a second if it permits only a more restricted set of messages, with the same outcome function.
  - In that case, the second mechanism is an *extension* of the first.
- A simplification has the *outcome closure property* if, when all players besides one (say, player j) report restricted messages, then any outcome player j could obtain by reporting any unrestricted message can be closely approximated for j by reporting some restricted message.

Example: Menu Auctions

- **Claim**: *The menu auction (aka "pay-as-bid package auction") restricted to additive bids satisfies the outcome closure property relative to the unrestricted menu auction.*
- The restricted version is a simultaneous sealed-bid auction
  - Bidders make separate bids for each item offered.
  - Each item is awarded to its highest bidder.
  - Bidder pays the sum of its winning bids.
- Outcome closure
  - Package bid wins against additive bids if it exceeds their sum
  - Same set and price could be accomplished by an additive bid with each component winning.

National Resident Matching Program

- **Claim**: *The Gale-Shapley mechanism restricted to responsive reports (as in the NRMP) satisfies the outcome closure property.*
- In the National Resident Matching Program,
  - doctors report rank-order lists of hospitals and hospitals report a number of openings and a rank-order lists of doctors.
  - the doctor-best stable assignment with respect to reported preferences is selected.
- Outcome closure
  - Any class achieved by a hospital by reporting any *extended* (substitutes) message is also achieved by ranking those students at the top in the restricted message.

Simplification Theorems

- **Theorem**. Let *u* be a profile of continuous utility functions and let $\varepsilon \geq 0$. If some report profile is a (full-information) ε -Nash equilibrium of a simplified mechanism satisfying the outcome closure property, then it is also a full-information ε -Nash equilibrium of the extended mechanism.
  - The case $\varepsilon = 0$ describes Nash equilibrium.

- **Theorem**. (Eduardo Perez, 2009): If a mechanism does not satisfy the outcome closure property, then there exists a profile of continuous preferences such that some Nash equilibrium of the simplified mechanism is not a Nash equilibrium of the extended mechanism.

Simplification and Equilibrium

- In models where longer reports incur additional cost and omitted value reports are treated as zeroes, simplification can sometimes *strictly and substantially improve* equilibrium performance.
- In such models, bad strict Nash equilibria are associated with
  - Coordination failures
  - Failures to make losing bids.

Google's Search Ads Auction

- Search advertising sold at auction
  - N ≥2 ad positions (higher positions worth strictly more)
  - M≥2 bidders
- Generalized Second Price Mechanism
  - ONE bid per bidder
  - Price is set by the just losing bid
  - Full information pure eqlm ➔ positive equilibrium revenue
- A "Natural" Extension
  - Each bidder may bid a separate price for each ad position
  - Sequence of second price auctions with winner elimination.
  - Full information pure eqlm ➔ zero equilibrium revenue

Rethinking Incentive Constraints

Incentives as Constraints (!?)

- *Incentive-compatible mechanisms can have very bad properties.*
  - In generic environments with (i) cash transfers, (ii) multi-dimensional signals, and (iii) interdependent values, a mechanism is *ex post* incentive-compatible if and only if its outcome is independent of all the signals. Jehiel, Meyer-ter-Vehn, Moldovanu and Zame (2006)*
  - Substituting *private values* for interdependent values, the unique package auction mechanism that is efficient, straightforward, and has zero payoffs for losing bidders is the *Vickrey* auction (Green and Laffont).
    - But it has problems related to low revenues, collusion, shill bidding and more.
  - For the course allocation problem, the unique efficient, incentive-compatible mechanism is *random serial dictatorship*, which can lead to terribly unfair outcomes.
- *Are there mechanisms with practically helpful incentive properties that avoid these difficulties?*

Vickrey Auction Has Multiple Flaws*

- Vickrey auctions can lead to *unacceptably low revenues* . . .
  - An example with ample competition but *zero* revenue:

Bidders	Item A	Item B	Pair AB
1	0	0	10
2	**10****	9.99	10
3	9.99	**10****	10

*Ausubel and Milgrom (2005), "The Lovely but Lonely Vickrey Auction."

More Flaws

- Vickrey auctions can lead to unacceptably low revenues, *promote false-name bids, lead sellers to disqualify bidders*…

Bidders	Item A	Item B	Pair AB
1	0	0	10
2	**10****	9.99	10
3	9.99	**10****	10

More Flaws

- Vickrey auctions can lead to unacceptably low revenues, promote false-name bids, lead sellers to disqualify bidders, *encourage collusion* and … more.

Bidders	Item A	Item B	Pair AB
1	0	0	**10****
2	4	3.99	4
3	3.99	4	4

"...but...but..."

- Isn't this analysis unfair? Don't real bidders have too little information to make such moves?

- Vickrey auctions are said to be straightforward, but *in a relevant expanded strategy space, they are not!*
  - Bidders can have incentives to bid under multiple identities.
  - Auctioneers can have incentives to exclude bidders.

Weaker Incentives: Package Bidding

- One idea (Day & Milgrom, 2007): How can one *minimize* the incentives to misreport, given that the outcome must be *core-selecting* (lie in the *core* with respect to reported values)?
- *Theorem*. A package auction minimizes the sum of bidders' maximum gains from deviations among core-selecting auctions if and only if it is a *minimizes revenues* on that set.
  - One-good example: second-price auction.
  - If goods for sale are *substitutes*, the Vickrey outcome is the unique minimum-revenue core outcome.
  - If goods are *not substitutes*, the Vickrey outcome need not lie in the core.

Equilibria of Core-Selecting Auctions

- Let π be a core *imputation* of the package auction setting – a vector of payoffs for individual participants.
- Consider the strategy profile in which each bidder n misreports its values, reducing them all by π_n. ("Truthful strategies, profit-target strategies, etc)

- *Theorem*. For every core-selecting package auction, the profile described above is a Simon-Zame (Nash) equilibrium profile and payoffs are given by π.

Another Approximate Approach

- Gains to deviants must vanish "in the limit" with replication.
  - Little or no incentive to misreport in settings with *many participants and items*.

Sample "Large Market" Results

- Roberts and Postlewaite (1976), "The Incentives for Price-Taking Behavior in Large Exchange Economies."
- Kojima and Pathak (2008), "Incentives and Stability in Large Two-Sided Matching Markets"
- Kojima and Minea (2009), "Incentives in the Probabilistic Serial Mechanism"
- Budish (2009), "The Combinatorial Assignment Problem: Approximate Competitive Equilibrium from Equal Incomes"

Connecting "Different" Products and Markets

Connections Among "Markets"

- Agents care not about items, but about *bundles* of items.
  - Example: securities trading
- Different products may be close substitutes
- Securities traders can link transactions only imperfectly by trading over time at posted prices.

- A new development in security markets
  - CBOE and *exact* trades
  - Transparency issues in practice

Connecting Substitutes

- When items are "strong substitutes" for all bidders
  - Integer competitive equilibrium allocations exist
  - Gale-Shapley matching algorithm yields stable/core allocations
  - Vickrey and Min Revenue Core auctions have same outcomes
  - Vickrey mechanism discourages false name bids, collusion among losers, and bidder exclusion
- …but *non-substitutes cases are hard*…
  - When possible preferences *strictly* include the set of substitutes preferences, the corresponding extended results are all false.

Easy Auctions for Substitutes

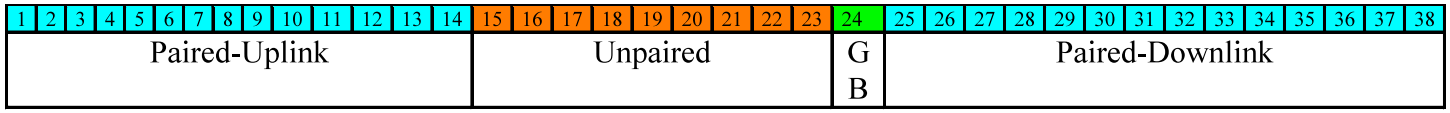
- Simultaneous multiple round (SMR) auction
  - Bidders call prices: Milgrom-Wilson-McAfee-McMillan (1993)
- SMR *clock* auctions
  - Ausubel (1996+…)
- Sealed-bid "assignment auction"
  - Milgrom (2009)

Mechanisms for General Cases

- Some theory research focuses on new mechanisms for non-substitutes cases, but experimenters still lead in this arena.
- "Experimentally tested" mechanisms
  - RAD
  - CCA
  - Plott mechanisms
  - UK auction mechanism
- …but *heterogeneous performance*

UK Band Planning

- Endogenous band plan and band conflation
- Sample Outcome: 9 unpaired and 14 paired lots.



UK Mechanism

- Research influencing the new UK spectrum auction.
  - Combinatorial clock auction (Porter-Rassenti-Roopnarine-Smith)
  - Clock proxy auction (Ausubel-Cramton-Milgrom)
  - Min-revenue core-selecting package auction (Day-Milgrom)
  - Revealed preference activity rule (Ausubel-Milgrom)
  - New computational methods (Day-Raghavan)

UK Auction Rules

- Two auction stages and three auction phases
  1. Principal stage assigns unspecific spectrum
     - Primary rounds: an ascending clock auction.
     - supplementary round: a direct mechanism which finds the total bid maximizing allocation and sets base prices equal to be the *Vickrey-nearest minimum-revenue core prices*.
  2. Assignment stage
     - A direct mechanism which finds the total bid maximizing assignment consistent with the principal stage and fixes "additional prices" to be the Vickrey-nearest minimum-revenue core prices.

Summary: Foundations Redux

- How Should Products/Contracts Be Defined?
- What Messages Should a Mechanism Use?
- How Should Incentives Be Provided?
- When (and How) Should "Different" Markets Be Linked?

End