The Promise and Problems of (Auction) Market Design

Paul Milgrom
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Market design is a kind of economic engineering, utilizing laboratory research, game theory, algorithms, simulations, and more. Its challenges inspire us to rethink longstanding fundamentals of economic theory.
Two Areas of Market Design

• Matching Markets without Money
  • Doctors & Hospitals
  • School assignments
  • Kidneys
  • Course allocation

• Auction Markets: Matching and Pricing and More
  • Radio spectrum
  • Power (electricity and gas)
  • “Commodities”
  • Internet advertising
Revisiting Foundations

- How Should Products/Contracts Be Defined?
  - “A commodity is characterized by its physical properties, the date at which it will be available, and the location at which it will be available.” (Debreu, 1959)

- When (and How) Should “Different” Markets Be Linked?
  - Always/never, as in General Equilibrium Theory?

- What Messages Should a Mechanism Use?
  - Revelation principle: “any equilibrium outcome of an arbitrary mechanism can be replicated by an incentive-compatible direct mechanism.” (2007 Nobel citation)

- How Should Incentives Be Provided?
  - Use “an incentive-compatible direct mechanism”?
Product Definitions
Product Definitions in Practice

- Wheat
  - From *The Book of Wheat* by Peter Dondlinger, published 1908: “…for each transaction they would analyze a sample to determine its value. The measurement costs were very high.”

- Diamonds
  - BHP Billiton auction: 19 “deals” are sold in “splits,” with “book” adjustments. (Cramton, Dinkin & Wilson, 2009)

- Radio spectrum auctions
  - Bandwidth, geographic area and …

- Advertising impressions
  - Keywords, interests, demographics, behavioral history, etc.
Effects of Product Definition

- Wheat example. Setting standards...
  - Reduced measurement costs (and/or adverse selection)
  - Reduced shipping cost (grain cars on trains)
  - Enabled futures markets for wheat
- …but finer classifications may lead to…
  - Better matching of goods to buyers
  - More efficient quality choices by suppliers
  - Thinner markets within each classification
- Online advertising examples
  - Facebook: Cubs stadium merchandise
  - Yahoo/McDonald’s “Happy Contract”
- Publishers’ fears of “commoditization”
Product definition questions bleed into message design issues.
Message Length Problem

- A direct mechanism requires reporting a value for every possible combination of licenses.
- In the US, FCC radio spectrum auctions may involve more than 1000 licenses.
  - Example – Auction 66: 1132 licenses
  - A report in such a mechanism conveys $2^{1132}$ numbers.
- Possible fixes?
  - Multi-round auctions.
  - Messages report only parameterized preferences.
Simplified Messages*

- Limited reporting changes the set of Nash equilibria.
  - Some equilibrium profiles may be eliminated, if the corresponding reports are eliminated by the simplification.
  - Some equilibrium profiles may be added, if all profitable deviations are eliminated by the simplification.
- A simplified mechanism avoids introducing new equilibria if it has the outcome closure property…

*Based on Milgrom (2009), “Simplified Mechanisms with an Application to Sponsored Search Auctions”
Outcome Closure Property (Formal)

Standard Set-up:

- Message profiles: \( M = M_1 \times \ldots \times M_N \)
- Outcome set is \( X \subseteq X_1 \times \ldots \times X_N \).
- A mechanism is \( \Omega = (M, \omega) \) with \( \omega : M \rightarrow X \).
- Agent \( j \)'s has utility payoff is \( u_j : X_j \rightarrow \mathbb{R} \).

New Definitions:

- Let \( M' \) be a subset of \( M \). Then, \( \Omega' = (M', \omega_{\mid M'}) \) is a simplification of \( \Omega = (M, \omega) \) and \( \Omega \) is an extension of \( \Omega' \).
- A simplification has the outcome closure property if for every player \( j \) and every profile of restricted messages \( m_{-j} \) for players \(-j\),
  \[ \text{cl}(\omega(M_j, m_j)) = \text{cl}(\omega(M'_{-j}, m_{-j})). \]
Again, in Ordinary English

- A mechanism \( \Omega = (M, \omega) \) is a pair consisting of a set of messages for each player and a function mapping messages to outcomes.
- A first mechanism is a *simplification* of a second if it permits only a more restricted set of messages, with the same outcome function.
  - In that case, the second mechanism is an *extension* of the first.
- A simplification has the *outcome closure property* if, when all players besides one (say, player \( j \)) report restricted messages, then any outcome player \( j \) could obtain by reporting any unrestricted message can be closely approximated for \( j \) by reporting some restricted message.
Example: Menu Auctions

- **Claim:** The menu auction (aka “pay-as-bid package auction”) restricted to additive bids satisfies the outcome closure property relative to the unrestricted menu auction.

- The restricted version is a simultaneous sealed-bid auction
  - Bidders make separate bids for each item offered.
  - Each item is awarded to its highest bidder.
  - Bidder pays the sum of its winning bids.

- Outcome closure
  - Package bid wins against additive bids if it exceeds their sum
  - Same set and price could be accomplished by an additive bid with each component winning.
National Resident Matching Program

- **Claim**: The Gale-Shapley mechanism restricted to responsive reports (as in the NRMP) satisfies the outcome closure property.

- In the National Resident Matching Program,
  - doctors report rank-order lists of hospitals and hospitals report a number of openings and a rank-order lists of doctors.
  - the doctor-best stable assignment with respect to reported preferences is selected.

- **Outcome closure**
  - Any class achieved by a hospital by reporting any *extended* (substitutes) message is also achieved by ranking those students at the top in the restricted message.
Simplification Theorems

• **Theorem.** Let $u$ be a profile of continuous utility functions and let $\varepsilon \geq 0$. If some report profile is a (full-information) $\varepsilon$-Nash equilibrium of a simplified mechanism satisfying the outcome closure property, then it is also a full-information $\varepsilon$-Nash equilibrium of the extended mechanism.
  
  • The case $\varepsilon = 0$ describes Nash equilibrium.

• **Theorem.** (Eduardo Perez, 2009): If a mechanism does not satisfy the outcome closure property, then there exists a profile of continuous preferences such that some Nash equilibrium of the simplified mechanism is not a Nash equilibrium of the extended mechanism.
Simplification and Equilibrium

- In models where longer reports incur additional cost and omitted value reports are treated as zeroes, simplification can sometimes strictly and substantially improve equilibrium performance.
- In such models, bad strict Nash equilibria are associated with:
  - Coordination failures
  - Failures to make losing bids.
Google’s Search Ads Auction

- Search advertising sold at auction
  - $N \geq 2$ ad positions (higher positions worth strictly more)
  - $M \geq 2$ bidders
- Generalized Second Price Mechanism
  - ONE bid per bidder
  - Price is set by the just losing bid
  - Full information pure eqlm $\Rightarrow$ positive equilibrium revenue
- A “Natural” Extension
  - Each bidder may bid a separate price for each ad position
  - Sequence of second price auctions with winner elimination.
  - Full information pure eqlm $\Rightarrow$ zero equilibrium revenue
Rethinking Incentive Constraints
Incentives as Constraints (!?)

- Incentive-compatible mechanisms can have very bad properties.
  - In generic environments with (i) cash transfers, (ii) multi-dimensional signals, and (iii) interdependent values, a mechanism is ex post incentive-compatible if and only if its outcome is independent of all the signals. Jehiel, Meyer-ter-Vehn, Moldovanu and Zame (2006)*
  - Substituting private values for interdependent values, the unique package auction mechanism that is efficient, straightforward, and has zero payoffs for losing bidders is the Vickrey auction (Green and Laffont).
    - But it has problems related to low revenues, collusion, shill bidding and more.
  - For the course allocation problem, the unique efficient, incentive-compatible mechanism is random serial dictatorship, which can lead to terribly unfair outcomes.
- Are there mechanisms with practically helpful incentive properties that avoid these difficulties?
Vickrey Auction Has Multiple Flaws*

- Vickrey auctions can lead to *unacceptably low revenues* …
- An example with ample competition but *zero* revenue:

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<td>1</td>
<td>0</td>
<td>0</td>
<td>10</td>
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<tr>
<td>2</td>
<td>10**</td>
<td>9.99</td>
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*Ausubel and Milgrom (2005), “The Lovely but Lonely Vickrey Auction.”*
More Flaws

- Vickrey auctions can lead to unacceptably low revenues, promote false-name bids, lead sellers to disqualify bidders…

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“...but...but...”

- Isn’t this analysis unfair? Don’t real bidders have too little information to make such moves?

- Vickrey auctions are said to be straightforward, but *in a relevant expanded strategy space, they are not!*
  - Bidders can have incentives to bid under multiple identities.
  - Auctioneers can have incentives to exclude bidders.
Weaker Incentives: Package Bidding

- One idea (Day & Milgrom, 2007): How can one minimize the incentives to misreport, given that the outcome must be core-selecting (lie in the core with respect to reported values)?

- **Theorem.** A package auction minimizes the sum of bidders’ maximum gains from deviations among core-selecting auctions if and only if it is a minimizes revenues on that set.
  - One-good example: second-price auction.
  - If goods for sale are substitutes, the Vickrey outcome is the unique minimum-revenue core outcome.
  - If goods are not substitutes, the Vickrey outcome need not lie in the core.
Equilibria of Core-Selecting Auctions

- Let $\pi$ be a core *imputation* of the package auction setting – a vector of payoffs for individual participants.
- Consider the strategy profile in which each bidder $n$ misreports its values, reducing them all by $\pi_n$. (“Truthful strategies, profit-target strategies, etc)  

\textit{Theorem}. For every core-selecting package auction, the profile described above is a Simon-Zame (Nash) equilibrium profile and payoffs are given by $\pi$. 

Another Approximate Approach

- Gains to deviants must vanish “in the limit” with replication.
  - Little or no incentive to misreport in settings with many participants and items.
Sample “Large Market” Results

Connecting “Different” Products and Markets
Connections Among “Markets”

- Agents care not about items, but about bundles of items.
  - Example: securities trading
- Different products may be close substitutes
- Securities traders can link transactions only imperfectly by trading over time at posted prices.

- A new development in security markets
  - CBOE and exact trades
  - Transparency issues in practice
Connecting Substitutes

- When items are “strong substitutes” for all bidders
  - Integer competitive equilibrium allocations exist
  - Gale-Shapley matching algorithm yields stable/core allocations
  - Vickrey and Min Revenue Core auctions have same outcomes
  - Vickrey mechanism discourages false name bids, collusion among losers, and bidder exclusion

- …but non-substitutes cases are hard…
  - When possible preferences strictly include the set of substitutes preferences, the corresponding extended results are all false.
Easy Auctions for Substitutes

• Simultaneous multiple round (SMR) auction

• SMR clock auctions
  • Ausubel (1996+…)

• Sealed-bid “assignment auction”
  • Milgrom (2009)
Mechanisms for General Cases

• Some theory research focuses on new mechanisms for non-substitutes cases, but experimenters still lead in this arena.

• “Experimentally tested” mechanisms
  • RAD
  • CCA
  • Plott mechanisms
  • UK auction mechanism

• …but heterogeneous performance
UK Band Planning

- Endogenous band plan and band conflation
- Sample Outcome: 9 unpaired and 14 paired lots.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Paired-Uplink | Unpaired | G | B | Paired-Downlink |
UK Mechanism

- Research influencing the new UK spectrum auction.
  - Combinatorial clock auction (Porter-Rassenti-Roopnarine-Smith)
  - Clock proxy auction (Ausubel-Cramton-Milgrom)
  - Min-revenue core-selecting package auction (Day-Milgrom)
  - Revealed preference activity rule (Ausubel-Milgrom)
  - New computational methods (Day-Raghavan)
UK Auction Rules

- Two auction stages and three auction phases
  1. Principal stage assigns unspecific spectrum
     - Primary rounds: an ascending clock auction.
     - supplementary round: a direct mechanism which finds the total bid maximizing allocation and sets base prices equal to be the Vickrey-nearest minimum-revenue core prices.
  2. Assignment stage
     - A direct mechanism which finds the total bid maximizing assignment consistent with the principal stage and fixes “additional prices” to be the Vickrey-nearest minimum-revenue core prices.
Summary: Foundations Redux

- How Should Products/Contracts Be Defined?
- What Messages Should a Mechanism Use?
- How Should Incentives Be Provided?
- When (and How) Should “Different” Markets Be Linked?
End