Trade and Information

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The Paper

Paul Milgrom and Nancy Stokey
"Information, Trade and Common Knowledge."

Journal of Economic Theory 26, 17-27 (1982).

No Trade Theorems: Informal Statement

Rational agents will not trade with each other on the basis of differences of information alone.

Conceptualizing Heterogeneous Beliefs

Historical Background:

1. $\rightarrow$60s: undifferentiated notion of heterogeneous beliefs; heterogeneous beliefs explain a lot....
2. 70s: Harsanyi's model of "incomplete information", theories of "asymmetric information" lead to realization that differences in information (under a "common prior assumption") and "differences in prior beliefs" are very different. No trade theorems chrystalize distinction.
3. 80s$\rightarrow$: distinction betweeen "asymmetric information" and "heterogeneous prior beliefs" as sources of different posterior beliefs internalized...

Some Important Features of the Milgrom-Stokey Statement

Combines some important features

1. Unified treatment of abstract trade, competitive markets (risk neutrality, risk aversion)
   - Kreps 77, Tirole 82, Holmstrom-Myerson 83
2. Higher order and common beliefs and knowledge at center stage
   - Aumann 76, Sebenius-Geanakoplos 83
3. Incomplete markets; only some differences in prior beliefs lead to trade
   - Morris 94

This Talk

1. Introduction
2. Review of Milgrom-Stokey 82
3. The Common Prior Assumption
4. Relaxing Common Knowledge
5. "Applications": Getting Around No Trade Theorems in Finance

Setting

1. Finite States of the World $\Omega = \Theta \times X$
   - Θ: Physical (payoff relevant) States
   - X: Signals (payoff irrelevant)
2. n traders; trader i described by
   - endowment $e_i : \Theta \to \mathbb{R}^l_+$
   - utility fn. $U_i : \Theta \times \mathbb{R}^l_+ \to \mathbb{R}$
   - prior $p_i \in \Delta(\Omega)$
   - partition $\widehat{P}_i$ of Ω

Common Prior Assumptions and Concordant Beliefs

Common Prior Assumption: for all θ, x

$$p_1(\theta, x) = p_2(\theta, x) = ... = p_n(\theta, x)$$

Concordant Beliefs: for all θ, x

$$p_1(x|\theta) = p_2(x|\theta) = ... = p_n(x|\theta)$$

"agreed interpretation of signals"

Abstract Trade

- A trade $t = (t_1, ..., t_n)$ where each $t_i : \Omega \rightarrow \mathbb{R}^l$
- A trade is feasible if
  1. $\sum_i t_i(\theta, x) \leq 0$ for all θ, x
  2. $e_i(\theta) + t_i(\theta, x) \geq 0$ for all i, θ, x
- A trade is a θ-trade if each t_i does not depend on x

Abstract No Trade Theorem

If....

1. traders are weakly risk averse,
2. e is Pareto-efficient relative to θ-trades
3. prior beliefs are concordant
4. common knowledge that θ-trade t is weakly preferred to no trade

Then each trader is indifferent between trade and no trade. Moreover, if traders are strictly risk averse, then t is the zero trade.

Proof

Follow Aumann 76:

1. if an event is common knowledge, it corresponds to an event in the meet of the traders' partitions;
2. if an agent is willing to trade on a common knowledge event, he would be willing to trade if the common knowledge event was the only thing he knew
3. if there are no symmetric information gains from trade, there is no trade

Compare:

1. if trade is a zero sum game, everyone cannot gain from trade (Kreps 77, Tirole 82)
2. interim efficient allocation $\Rightarrow$ no common knowledge agreement to move to alternative allocation (Holmstrom-Myerson 83)

No Trade and Rational Expectations Equilibrium

Suppose all traders strictly risk averse, e is efficient prior to observation of signals and supported by price vector $q : \Theta \longrightarrow \mathbb{R}_{+}^{l}$. After observation of signals, e is still an equilibrium with prices $\widehat{q} : \Theta \times X \longrightarrow \mathbb{R}_{+}^{l}$.

$$p_i\left(\theta | P_i\left(\omega\right), \widehat{q}\right) = p_i\left(\theta | \widehat{q}\right)$$

Justifying the Common Prior Assumption

Harsanyi Doctrine:
"Differences in Beliefs are explained by Differences in Information"

what could this mean?

Asymmetric Information Perspective

- There was an "ex ante stage".
- In this ex ante stage, traders had identical beliefs about everything (including the distribution of signals they would observe in the future). Then they observe private signals.....
- In this setting, the common prior assumption = "Differences in Beliefs are explained by Differences in Information"
- Meaningful assumption not entailed by economists' traditional interpretation of rationality (Morris 95).

Incomplete Information Perspective

- There is not an "ex ante stage".
- "Types" are a convenient device to summarize possible beliefs and higher order beliefs about payoff relevant states, as proposed by Harsanyi 67/68 and later formalized by Mertens-Zamir 85.
- Gul 98: we cannot attach a meaning (empirical or conceptual) to the statement "Differences in Beliefs are explained by Differences in Information" under this interpretation.

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- But can we at least give a meaningful interpretation of the common prior assumption without appeal to a counterfactual ex ante stage?

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- But can we at least give a meaningful interpretation of the common prior assumption without appeal to a counterfactual ex ante stage?
- There is good news and bad news.

What is the Common Prior Assumption without a Prior Stage?

- Ex Ante Definition of Common Prior Assumption: for all ω

$$p_1(\omega) = p_2(\omega) = ... = p_n(\omega)$$

- Interim Definition of the Common Prior Assumption (Harsanyi Consistency): there exists a prior p^* such that i and ω,

$$p_i\left(\omega \left| \widehat{P}_i(\omega)\right.\right) = p^*\left(\omega \left| \widehat{P}_i(\omega)\right.\right)$$

The Good News

- Feinberg 00 provides a syntactic characterization of the common prior assumption using posterior beliefs alone
- Restricted to compact type spaces, with objective coin tosses added to the natural language
- see also Bonanno-Nehring 99

The Bad News

Feinberg 00 shows that a type is a common prior type if and only if he is sure that there cannot be common knowledge that he disagrees with others about something.

The Bad News

(1) MILGROM-STOKEY. If the common prior assumption holds, there cannot be common knowledge of willingness to trade.
(2) FEINBERG. The common prior assumption holds if and only if there cannot be common knowledge of willingness to trade.

Substituting (2) into (1): If there cannot be common knowledge of willingness to trade, then there cannot be common knowledge of willingness to trade.

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Feinberg using generalization of finite state space no trade theorem converse of Morris 94 (with asymmetric information interpretation)

Higher Order Expectation Characterization of the Common Prior Assumption

Samet 98:

- Fix a random variable $\widetilde{X}$.
- Let x_1 be 1's expectation of $\widetilde{X}$
- Let x_2 be 2's expectation of 1's expectation of $\widetilde{X}$
- Let x_3 be 1's expectation of 2's expectation of 1's expectation of $\widetilde{X}$
- etc....

Common prior assumption satisfied if and only if sequence $(x_k)_{k=1,2,..}$ converges.

- Lipman 03: "Finite Order Implications of Common Priors." There are none.

Operational Distinction between "Differences in Information" and "Differences in Priors"

- Differences in Information Alone: $(x_k)_{k=1,2,..}$ converges
- Differences in Priors Alone: $(x_k)_{k=1,2,..}$ two-cycle
- Information: non-two cycle
- Non-Common Priors: non-convergence
- Interesting case: between the two

Common Prior Assumption Conclusion

- Common Prior Assumption needs more examination.
- It is inextricably linked to common knowledge assumptions about enviroment.
- Every applied economist is making critical assumptions about "tails of beliefs" or, equivalently, what is commonly understood.

Approximate Common Knowledge

- An event is common knowledge if everyone knows it, everyone knows that everyone knows it, and so on ad infinitum.
- An event is approximate common knowledge if.....
  1. everyone knows it, everyone knows that everyone knows it, and so on n times, for large n [nth level mutual knowledge: e.g., Rubinstein 89, corresponds to closeness of types in the universal type space in the product topology....]
  2. everyone p-believes it, everyone p-believes that everyone p-believes it, and so on ad infinitum, for p close to 1 [common p-belief: Monderer-Samet 89]
- Largish game theory literature points out that (1) does not generate continuity in strategic outcomes, while (2) does.

Approximate Common Knowledge and No Trade Theorems

- Milgrom-Stokey 82 show nth level mutual knowledge (1) does not imply no trade theorem (c.f. Geanakoplos-Polemarchakis 82)
- With some nuances, a small literature shows that common p-belief (2) is enough for approximate no trade theorems.
- Geanakoplos 94, Sonsino 95, Neeman 96, Morris 99.

Reversal

- No Trade Theorem highlights importance of common knowledge for trade
  - (Approximate) common knowledge of no gains from trade $\Rightarrow$ no trade
- Converse?
  - (Approximate) common knowledge of gains from trade $\Rightarrow$ trade
  - c.f. Myerson-Sattherwaite 83
- Morris-Shin 09:
  - converse result in special setting
  - higher order belief assumptions should be taken seriously in this context

Contagious Adverse Selection

- Buyer values an object at $v + c$, seller values it at $v - c$
- With high probability $\frac{1-\delta}{1+\delta}$, $v = \overline{v}$ and neither agent is informed
- With small probability $\frac{\delta}{1+\delta}$, $v = \overline{v} - M$ and the seller only is informed
- With small probability $\frac{\delta}{1+\delta}$, $v = \overline{v} + M$ and the buyer only is informed
- Trade at price $\overline{v}$
- Define the "loss ratio" $\psi = \frac{\delta(M-c)}{(1-\delta)c} \approx \frac{\delta M}{c}$

Common Knowledge

- If there is common knowledge of ψ, then there is a trade equilibrium if and only if $\psi \leq 1$.
- Gains from trade $2c$

Relaxing Common Knowledge

- Suppose there is uncertainty about the loss ratio ψ.
- Write E for the event that each buyer expects uninformed seller to trade
- Expected Payoff to buyer from trading:

$$(1-\delta)\Pr(E)\,c+\delta(c-M)$$

- Trade is Best Response if

$$\Pr(E)\geq\frac{\delta(c-M)}{(1-\delta)\,c}=\psi$$

Relaxing Common Knowledge

- Necessary and sufficient conditions for trade:
  1. Each (uninformed) trader's expectation of ψ is less than 1
  2. Each trader believes (1) with probability at least equal to his expectation of ψ
  3. Each trader believes (2) with probability at least equal to his expectation of ψ
  4. etc....

Relaxing Common Knowledge

- Suppose there is uncertainty about the loss ratio ψ.
- Necessary and sufficient conditions for trade:
  1. Each (uninformed) trader's expectation of ψ is less than 1
  2. Each trader believes (1) with probability at least equal to his expectation of ψ
  3. Each trader believes (2) with probability at least equal to his expectation of ψ
  4. etc....
- Common ψ-belief: generalization of common p-belief with "p" varying with the trader and state and equal to a trader's expectation of ψ

Relaxing Common Knowledge

- "Market Confidence" $=$ common ψ-belief
- Equivalent fixed point characterization: does there exist an event E such that whenever that event is true, everyone assigns probability at least equal to his expectation of ψ to E?
- Ex ante gains from trade $= 2c \times \Pr(\text{Market Confidence})$
- Extends to many traders;
    - generalized belief operator
        - each trader's expectation of proportion believing E is greater than his expectation of ψ

No Trade in Practise: Toxic Assets Fall 08

- Shutdown of markets for asset backed securities
- Many reasons:
  - perverse institutional incentives
  - wipe out of capital of natural buyers
  - adverse selection
- Why did adverse selection "blow up"?

Toxic Assets

- Shutdown of markets for asset backed securities
- Many reasons:
  - perverse institutional incentives
  - wipe out of capital of natural buyers
  - adverse selection
- Used to be approximate common knowledge that most people would treat AAA securities as perfect substitutes: market confidence
- Loss of confidence (= approximate common knowledge) is enough to break down trade even in assets that have not suffered obvious losses.

Relaxing Common Knowledge Conclusion

- Thinking about the role common knowledge assumptions in no trade theorems helps understand financial markets
- Financial Markets require some common understanding of features of market
- Loss of "market confidence" = loss of that common understanding
- Accounting/disclosure should be geared to generating common knowledge (Morris-Shin 07)

The Top Four

- Glosten-Milgrom 85 "Bid, Ask and Transactions Prices in a Specialist Market...." [2416]
- De Long-Shleifer-Summers-Waldman 90 "Noise Trader Risk in Financial Markets" [1911]
- Scharfstein-Stein 90 "Herd Behavior and Investment" [1285]
- Black 86 "Noise" [1286]

Must provide a non-informational reason for trade.

What should no trade theorems have told us about modelling financial markets?

THEORETICAL INSIGHT 1: It is rational to draw inferences from others' willingness to trade.
THEORETICAL INSIGHT 2: Under the common prior assumption, trade requires non-belief motives.
EMPIRICAL INSIGHT: Volume of trade on financial markets too large to be explained without belief motives.

- So we need to relax the common prior assumption in our models of financial markets.
- But how to relax them? Can "anything" happen (Morris 95)?
- Perhaps we should look to psychology for origins of beliefs, but keep theoretical insight 1?

What's actually happened?

Bifurcation:

- Asymmetric Information Literature (market microstructure, insider trading)
- Behavioral Finance ($\approx$relaxing the common prior assumption under symmetric information)

Over-Confidence

"overestimation of the precision of one's information"

Over-Confidence Example

- Two states, $\Omega = \{L, H\}$, equally likely.
- Alice only observes one of two signals, $S = \{l, h\}$. Bob observes nothing.
- Trader i thinks Alice's signal is "correct" with probability $q_i \in \left[\frac{1}{2}, 1\right]$.
- Prior beliefs are: Thus Alice has prior beliefs

Alice	l	h
L	$\frac{1}{2}q_A$	$\frac{1}{2}(1-q_A)$
H	$\frac{1}{2}(1-q_A)$	$\frac{1}{2}q_A$

Bob	l	h
L	$\frac{1}{2}q_B$	$\frac{1}{2}(1-q_B)$
H	$\frac{1}{2}(1-q_B)$	$\frac{1}{2}q_B$

Over-Confidence I

- Alice (relatively) "over-confident" if $q_A > q_B$, e.g., $q_A = \frac{3}{4}$ and $q_B = \frac{2}{3}$ giving priors

Alice	l	h
L	$\frac{3}{8}$	$\frac{1}{8}$
H	$\frac{1}{8}$	$\frac{3}{8}$

Bob	l	h
L	$\frac{1}{3}$	$\frac{1}{6}$
H	$\frac{1}{6}$	$\frac{1}{3}$

- Overconfidence will lead to trade.
- Consider the game:
    - Alice makes a prediction about L or H.
    - If the prediction is correct, Bob pays Alice \$24.
    - If the prediction is incorrect, Alice pays Bob \$60.

Over-Confidence II

- Consider the game:
  - Alice makes a prediction about L or H.
  - If the prediction is correct, Bob pays Alice \$24.
  - If the prediction is incorrect, Alice pays Bob \$60.
- Expected gain to Alice: $\frac{3}{4}(24) + \frac{1}{4}(-60) = 3$
- Expected gain to Bob: $\frac{2}{3}(-24) + \frac{1}{3}(60) = 2$
- Gain to Truth-Telling:
  $3 = \frac{3}{4}(24) + \frac{1}{4}(-60) \geq \frac{1}{4}(24) + \frac{3}{4}(-60) = -39$

Underconfidence I

- Underconfidence with $q_A < q_B$; e.g., $q_A = \frac{2}{3}$, $q_B = \frac{3}{4}$ gives priors

Alice	l	h
L	$\frac{1}{3}$	$\frac{1}{6}$
H	$\frac{1}{6}$	$\frac{1}{3}$

Bob	l	h
L	$\frac{3}{8}$	$\frac{1}{8}$
H	$\frac{1}{8}$	$\frac{3}{8}$

- Consider the game as before:
  - Alice makes a prediction about L or H.
  - If the prediction is correct, Alice pays Bob \$24.
  - If the prediction is false, Bob pays Alice \$60.

Underconfidence II

- Consider the game as before:
    - Alice makes a prediction about L or H.
    - If the prediction is correct, Alice pays Bob \$24.
    - If the prediction is false, Bob pays Alice \$60.
- Expected gain to Alice: $\frac{2}{3}(-24)+\frac{1}{3}(60)=2$
- Expected gain to Bob: $\frac{3}{4}(24)+\frac{1}{4}(-60)=3$
- Gain to Truth-Telling:
  $2=\frac{2}{3}(-24)+\frac{1}{3}(60)<\frac{2}{3}(60)+\frac{1}{3}(-24)=32$

Overconfidence Example

- Write a^* and b^* for agents' unconditional probabilities of L and a_s and b_s for their probabilities conditional on signal s.
- Assume w.l.o.g. that $a_l > a_h$.
- Then no trade if

$$b_l \geq a_l \geq b^* \geq a_h \geq b_h$$

- over confidence gives trade, under confidence doesn't
- Morris 94: incentive compatibility gives no trade even with differences in prior beliefs
- third relaxation in talk: concordance, Harsanyi consistency + this (under confidence)

Overconfidence in Behavioral Finance

- words: "overestimation of the precision of one's information"
- models: overconfidence = difference in priors in symmetric information model (e.g., Odean 98, Scheinkman-Xiong 03)
- exception: Kyle-Wang 97; but no opportunity to trade on different precisions
- important to understand that what is driving theoretical/empirical results on overconfidence

Finance Conclusions

- Time to re-integrate information based and prior based differences in beliefs
- Model noise traders in more integrated way
- Add asymmetric information to behavioral finance

Trade and Information Conclusions

- Milgrom-Stokey 82 and no trade theorems crystalized importance of (1) conceptual distinction between asymmetric information and heterogeneous prior beliefs; and (2) common knowledge.
- Need to be self-aware of how common prior is being used.
- No trade theorems teach us the importance of common understanding (= market confidence)
- Time to re-integrate information based and prior based differences in beliefs in finance models