Markets with Multidimensional Private Information*

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Abstract

This paper explores price formation in environments with multidimensional private information. Asset sellers are informed both about their need to raise cash and about the quality of the asset they are selling. Asset buyers have rational expectations about the distribution of assets for sale at different prices. Any equilibrium with trade involves partial pooling: identical assets sell for different prices, depending on the seller's need to raise cash; while conversely different assets sell for the same price. Sellers who set a higher price are less likely to succeed at selling. We find a simple condition under which a continuum of such equilibria exist. This condition admits the possibility that some assets are intrinsically worthless, in which case there is also an equilibrium with no trade. In general, the set of equilibria depends on the joint distribution of seller and asset characteristics, and not just the support of that distribution.

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1 Introduction

This paper develops a model of trade in an asset market with adverse selection. The economy is populated by a measure 1 of risk-neutral investors who live for two periods. At the beginning of period 1, each investor is endowed with one unit of perishable fruit and one tree that produces fruit in period 2. Trees are heterogeneous in terms of the amount of fruit δ they produce in period 2. Investors are heterogenous in their discount factor β. At the beginning of the period each investor observes the quality of his tree (the amount of fruit it will produce) and his discount factor. Next, there is trade of fruit for trees. Investors may use their fruit to buy trees, sell their tree for fruit, engage in both activities, or simply live in autarky. We allow investors to buy or sell at any price, forming beliefs about the probability that they will be able to trade at that price and about the composition of trees offered for sale at that price. Trade is rationed by the short side of the market at any price, with all traders on the long side of the market equally likely to be trade.

Our goal is to characterize the set of equilibria of this model. Towards that end, we define a key model primitive, the expected quality of a tree δ conditional on the owner's continuation value v, the product of the owner's discount factor β and the tree quality δ. We assume throughout our analysis that this function is continuous and strictly increasing in v. Under this parameter restriction, we prove that an equilibrium always exists. Moreover, if the minimum tree quality is equal to the average tree quality among investors with the lowest continuation value, then the equilibrium allocation is unique. On the other hand, if the minimum tree quality is smaller, then we prove that a continuum of equilibria exists, distinguished by the payoffs of different investors.

The set of equilibrium payoffs is parameterized by a single number, the lowest probability (or highest price) at which an investor with the lowest continuation value is willing to sell her tree. If the minimum tree quality is positive and equal to the average tree quality for investors with the lowest continuation value, then investors with the lowest continuation value sell their tree with probability 1. If the minimum tree quality is zero and the average tree quality is positive, then investors with the lowest continuation value may sell their tree with any probability between 0 and 1, depending on the equilibrium. If both the minimum and average are positive, then there is a positive lower bound on the sale probability, while if both are zero then investors with the lowest continuation value do not sell their tree.

In any equilibrium in which investors with the lowest tree quality sell their tree with positive probability, some investors with higher continuation values sell their trees, but at higher price and with a lower probability. On the other hand, all investors with the same continuation value (except possibly those with the lowest continuation value) sell their trees at the same price. As a result, an investor is uncertain about exactly what quality tree he will purchase at a given price. Conversely, if investors with the lowest quality tree do not sell, then there is no trade in equilibrium.

Our model suggests two notions of how adverse selection can generate a crisis episode in which trade collapses. The first is that the joint distribution of tree quality and asset holdings changes so as to reduce the liquidity in any equilibrium. At an extreme, if the expected tree quality of an investor with the lowest continuation value is zero, then all trade must collapse. The second is a shift in the equilibrium holding fixed the exogenous parameters. We argue that a shift to an equilibrium in which the investor with the lowest continuation value trades with a lower probability might represent a buyers' strike. In a such an equilibrium, all investors set higher sale prices, some investors stop buying trees, and liquidity, as measured either by the volume of fruit or the volume of trees that changes hands, declines.

The equilibria of this model with multidimensional private information differ from our previous work in which investors' discount factors are observable (Guerrieri and Shimer, 2012). In that model, we found that there is a unique fully separating equilibrium and that trees of higher quality trade at higher price in less liquid markets. It is worth highlighting four dimensions in which the predictions of the two models differ. First, with multidimensional private information there is price dispersion for trees of the same quality and heterogeneous trees selling for the same price. In our prior work, there was a one-to-one mapping from tree quality to price. Second, the set of equilibrium payoffs in this paper is affected by the joint distribution of discount factors and tree quality, while in our prior work, equilibrium payoffs only depended on the support of the distribution and the relative supply of fruit. Third, with multidimensional private information some investors both buy and sell trees. In contrast, with private information only about tree quality, investors only participate on one side of the market. Finally, we find conditions under which a continuum of equilibria exist, while in our previous work, the equilibrium was unique.

Our notion of equilibrium builds on Guerrieri, Shimer and Wright (2010). To our knowledge, Chang (2012) is the only previous paper that has explored multidimensional private information in that sort of environment. There are several important differences between the results in the two papers. First, Chang looks at an environment in which the role of an investor as a buyer or seller is determined exogenously. We allow investors to choose whether to buy trees, sell trees, do both, or do neither, an important possibility in more realistic environments. Second, Chang assumes that sellers are heterogeneous while buyers are homogeneous. Moreover, all buyers value any asset more than any seller does, so all trades are socially beneficial. This ensures that in equilibrium, all assets are sold with a positive probability. In our model, investors are heterogeneous, the decision to buy and sell is endogenous, and in equilibrium some trees are transferred from investors who value them more to investors who value them less. As a result, we find that some investors choose not to attempt to sell their trees in equilibrium.[1] Third, under an analogous parameter restriction to ours, Chang only characterizes one equilibrium, while we prove that in our economy there can generically be a continuum of equilibria. Fourth, Chang (2012) characterizes equilibria when the parameter restriction fails. The current version of our paper does not do this.

Numerous previous papers (e.g. Eisfeldt, 2004; Kurlat, 2009; Daley and Green, 2010; Chari, Shourideh and Zetlin-Jones, 2010; Chiu and Koeppl, 2011; Tirole, 2012) have developed models of adverse selection in which all trade occurs at a single price. Those papers do not allow investors to consider trading at a different price. In contrast, this paper builds on our previous work, (Guerrieri, Shimer and Wright, 2010; Guerrieri and Shimer, 2012), allowing trade at any price but recognizing that sellers who demand a high price may be rationed. This implies that in any equilibrium with trade, trade occurs at a range of prices.

The paper proceeds as follows. Section 2 lays out the basic model. We analyze the equilibrium when there is symmetric information in Section 3. We then consider a one price equilibrium in Section 4. Finally, we turn to our main model with multidimensional private information in Section 5. We characterize the equilibrium through a sequence of lemmas and establish the conditions under which a continuum of equilibria exist. Section 6 concludes with a brief discussion of whether some of the equilibria can be understood as buyers' strikes.

2 Model

The economy lasts for two periods, $t = 1, 2$. It is populated by a unit measure of risk-neutral investors with heterogeneous discount factors $\beta \in \mathbb{B} \subseteq \mathbb{R}_+$. Each investor is endowed in period 1 with one unit of fruit and one tree that bears fruit in period 2. Trees are heterogenous in the amount $\delta \in \mathbb{D} \subseteq \mathbb{R}_+$ of fruit they produce. Both fruit and trees are divisible. Fruit is perishable and must be consumed within the period and consumption must be nonnegative in each period.

At the beginning of period 1, each investor observes his type, that is, his discount factor β. and the quality of his tree δ. Next, there is a market in which fruit and trees are exchanged. Each investor makes independent buying and selling decisions and so may engage in trade on both sides of the market, one side, or none. We impose a "fruit-in-advance constraint": an investor can only buy trees using the fruit he holds at the beginning of the period and

[1] Formally we model this as investors setting a high price at which they know they will be unable to sell their tree.

so must consume any fruit he gets from selling his tree.[2] After the market meets, investors consume any remaining period 1 fruit, c_1. In period 2, each investor consumes the fruit produced by the trees he holds in that period, c_2. An investor with discount factor β seeks to maximize $\mathbb{E}(c_1 + \beta c_2)$, where expectations recognize that the investor may be uncertain about the whether he will succeed in buying and selling trees and about the quality of the trees that he buys.

Let $G : \mathbb{B} \times \mathbb{D} \mapsto [0, 1]$ denote the initial joint distribution of discount factors and endowed tree quality, so $G(\beta, \delta)$ is the measure of individuals who have a discount factor no more than β and are endowed with a tree bearing no more than δ fruit. We assume G is atomless, that its support is convex, and let g denote the associated density. Formally, an individual (β, δ, i) is a discount factor $\beta \in \mathbb{B}$, a tree quality $\delta \in \mathbb{D}$, and a name $i \in [0, g(\beta, \delta)]$. Because there are many individuals, we ignore mixed strategies throughout this paper.

We analyze three different versions of this model. First, we study a benchmark model with symmetric information. Second, we analyze a market structure in which all transactions must take place at a single price. Third, we study a model where investors are privately informed about both their tree quality and their discount factor, but they are allowed to choose different prices.

3 Symmetric Information

Let us start by introducing the benchmark economy where information is complete. All investors observe the quality of all trees and the discount factor of all investors. In this environment, we look for the competitive equilibrium.

In a competitive equilibrium, different trees sell for different prices. An equilibrium is then described by a price schedule $p : \mathbb{D} \mapsto R_+$, where $p(\delta)$ denotes the price of a tree of type δ. Each investor (β, δ, i) takes this price schedule as given and decides whether to sell their tree, whether to use their fruit to buy a tree, and what type of tree to buy. Let $\mathbb{I}_s(\beta, \delta, i)$ be an indicator function which takes value 1 if (β, δ, i) sells his tree. Let $d_b(\beta, \delta, i)$ denote the type of tree that (β, δ, i) buys using his fruit, with $d_b(\beta, \delta, i) = \varnothing$ indicating that the investor consumes his fruit. Then a competitive equilibrium satisfies the following three conditions:

Definition 1 *An equilibrium is given by functions* $p : \mathbb{D} \mapsto \mathbb{R}_+$, $\mathbb{I}_s : \mathbb{B} \times \mathbb{D} \times \mathbb{R}_+ \mapsto \{0, 1\}$, *and* $d_b : \mathbb{B} \times \mathbb{D} \times \mathbb{R}_+ \mapsto \mathbb{D} \cup \varnothing$ *where the functions satisfy the following conditions:*

[2] Other assumptions are possible here. While they would change some of our calculations, relaxing this constraint would not alter our main results.

1. *Optimal Selling Decision: given* $p(\delta)$*, for all* (β, δ, i)

$$\mathbb{I}_s(\beta, \delta, i) = \begin{cases} 1 & \\ & p(\delta) \gtrless \beta\delta; \\ 0 & \end{cases}$$

2. *Optimal Buying Decision: given* $p(\delta)$*, for all* (β, δ', i)*,*

$$d_b(\beta, \delta', i) \in \arg\max_{\delta \in \mathbb{D}} \frac{\delta}{p(\delta)}$$

   if $\max_{\delta \in \mathbb{D}} \beta\delta/p(\delta) > 1$ *and* $d_b(\beta, \delta', i) = \varnothing$ *otherwise;*

3. *Market Clearing: for each* δ*,* $p(\delta)\, d\mu_s(\delta) = d\mu_b(\delta)$*, where*

$$\mu_s(\delta) \equiv \int_0^\delta \int_{\mathbb{B}} \int_0^{g(\beta,\delta')} \mathbb{I}_s(\beta, \delta', i)\, di\, d\beta\, d\delta$$
$$\mu_b(\delta) \equiv \int_{d_b(\beta,\delta',i) \leq \delta} \int_{\mathbb{B}} \int_0^{g(\beta,\delta')} di\, d\beta\, d\delta'$$

   are the measure of trees worse than δ *for sale and the measure of fruit used to buy trees worse than* δ*, respectively.*

The first condition requires that (β, δ, i) sells his tree if the price exceeds the value he places on holding onto the tree, $\beta\delta$. The second condition requires that (β, δ', i) buys a tree of type δ if this is the tree with the highest dividend-price ratio and if his discount factor exceeds the price-dividend ratio. This follows immediately from the tradeoff between consuming his unit of fruit or using it to buy $1/p(\delta)$ type δ trees. The final equilibrium condition implies that the amount of fruit used to purchase trees with dividends δ is equal to the product of the number of such trees offered for sale and the sale price, so the market for each type of tree clears.

It is straightforward to prove that the price function must satisfy $p(\delta) = \hat{\beta}\delta$ for some $\hat{\beta} \in [0, 1]$. All investors with discount factor $\beta > \hat{\beta}$ use their fruit to buy any type of tree (since all have the same price-dividend ratio) and do not sell their tree. All investors with discount factor $\beta < \hat{\beta}$ sell their tree.

The market clearing condition determines the threshold $\hat{\beta}$. With the characterization in the previous paragraph, this reduces to the following single market clearing condition:

$$\int_{\mathbb{D}} \int_{\hat{\beta}}^{\infty} g(\beta, \delta)\, d\beta\, d\delta = \hat{\beta} \int_{\mathbb{D}} \int_0^{\hat{\beta}} \delta g(\beta, \delta)\, d\beta\, d\delta. \tag{1}$$

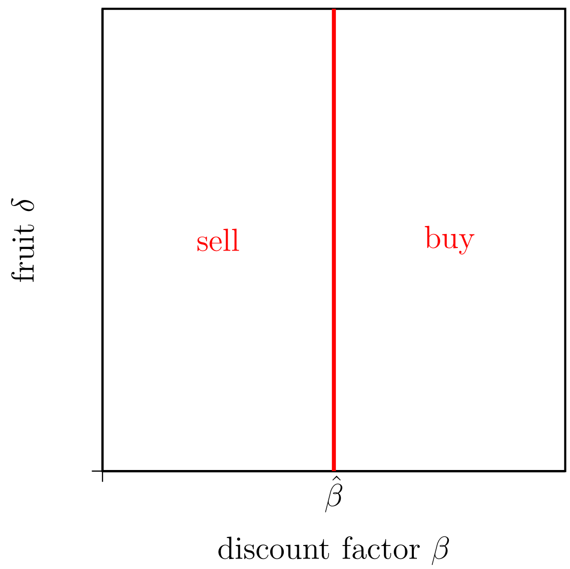

Figure 1: Competitive equilibrium with symmetric information

The left hand side is the fruit held by patient investors while the right hand side is the cost of purchasing the trees held by the impatient investors.

We summarize this analysis in the next proposition:

Proposition 1 *In a competitive equilibrium, $p(\delta) = \hat{\beta}\delta$ for all δ, where $\hat{\beta}$ solves 1. Moreover, any investor (β, δ, i) sells his tree if and only if $\beta < \hat{\beta}$ and uses his fruit to buy any tree for sale otherwise.*

Figure 1 represents the competitive equilibrium under symmetric information in the space (β, δ). The equilibrium can be characterized by the cutoff $\hat{\beta}$ so that all the investors with $\beta > \hat{\beta}$ always buy and $\beta < \hat{\beta}$ always sell irrespective of the type of tree that they have.

4 One Price Equilibrium

Much of the literature on adverse selection in financial markets assumes that all trade occurs at a common price p, so the equilibrium is *pooling* (see, for example, Eisfeldt, 2004; Kurlat, 2009; Daley and Green, 2010; Chari, Shourideh and Zetlin-Jones, 2010). In such papers, the environment is set up in such a way that a seller cannot even consider selling his trees at a price different than p. In this section, following this tradition, we consider a definition of equilibrium where we restrict all trades to occur at a common price p. Investors must decide whether to sell their trees at this price, knowing the quality of their tree. They also must decide whether to use their fruit to buy trees at this price, knowing only that someone offered it for sale at this price. The rest of the environment is exactly as in our benchmark model.

Again let $\mathbb{I}_s(\beta, \delta, i)$ indicate whether (β, δ, i) sells his tree. The decision of to buy trees is simpler and can now also be reduced to an indicator $\mathbb{I}_b(\beta, \delta, i)$. Then a one price equilibrium satisfies four conditions:

Definition 2 *A one price equilibrium is a price $p \in \mathbb{R}_+$, an expected tree quality $\Delta \in \mathbb{D}$, and functions $\mathbb{I}_s : \mathbb{B} \times \mathbb{D} \times \mathbb{R}_+ \mapsto \{0, 1\}$ and $\mathbb{I}_b : \mathbb{B} \times \mathbb{D} \times \mathbb{R}_+ \mapsto \{0, 1\}$ satisfying the following conditions:*

1. *Optimal Selling Decision: given p, for all (β, δ, i)*

$$\mathbb{I}_s(\beta, \delta, i) = \begin{cases} 1 & \text{if } p \gtrless \beta\delta; \\ 0 & \end{cases}$$

2. *Optimal Buying Decision: given p and Δ, for all (β, δ, i)*

$$\mathbb{I}_b(\beta, \delta, i) = \begin{cases} 1 & \text{if } \beta\Delta \gtrless p; \\ 0 & \end{cases}$$

3. *Beliefs:*

   (a) *if there exists a (β, δ, i) such that $\mathbb{I}_s(\beta, \delta, i) = 1$, $\Delta = \mathbb{E}(\delta | \mathbb{I}_s(\beta', \delta', i') = 1)$;*

   (b) *otherwise $\Delta \in \mathbb{D}$;*

4. *Market Clearing:*

$$\int_{\mathbb{B}} \int_{\mathbb{D}} \int_0^{g(\beta,\delta)} \left(\mathbb{I}_b(\beta, \delta, i) - p\mathbb{I}_s(\beta, \delta, i)\right) di\, d\delta\, d\beta = 0$$

The first equilibrium condition is essentially unchanged from the competitive equilibrium. An investor sells his tree if and only if the price exceeds his continuation value of holding on to the tree, $p > \beta\delta$. The second condition states that an investor uses his fruit to buy trees if his discount factor times the expected quality of the tree he would buy, Δ, exceeds exceeds the price he would pay. The third condition determines Δ, beliefs about the quality of tree purchased. Assuming that in equilibrium at least one investor sells his tree, Δ is given by the expected tree quality of the trees that are sold; otherwise it is only restricted to lie in the the set of available tree qualities. The final condition again imposes that the fruit market clears, so the fruit that investors use to buy trees is equal to the cost of buying the trees sold by other investors.

Once again, there is a marginal investor $\hat{\beta}$ who is just indifferent about buying trees, $p = \hat{\beta}\Delta$. Using the first and third parts of the definition of equilibrium, this reduces to

$$p = \hat{\beta}\frac{\int_{\mathbb{D}} \int_0^{p/\delta} \delta g(\beta, \delta)\, d\beta\, d\delta}{\int_{\mathbb{D}} \int_0^{p/\delta} g(\beta, \delta)\, d\beta\, d\delta}. \tag{2}$$

Finally, since all individuals more patient that $\hat{\beta}$ buy trees while all trees held by individuals with $\beta\delta < p$ are sold, the fruit market clearing condition reduces to

$$\int_{\mathbb{D}} \int_{\hat{\beta}}^{\infty} g(\beta, \delta)\, d\beta\, d\delta = p \int_{\mathbb{D}} \int_0^{p/\delta} g(\beta, \delta)\, d\beta\, d\delta. \tag{3}$$

This is summarized in the next proposition.

Proposition 2 *Any pair* $(p, \hat{\beta})$ *that satisfies equations* (2) *and* (3) *is a one price equilibrium.* $I_s(\beta, \delta, i) = 1$ *iff* $\beta\delta < p$ *and* $I_b(\beta, \delta, i) = 1$ *iff* $\beta > \hat{\beta}$*, while* $\Delta = p/\hat{\beta}$*.*

Figure 2 represents a one price equilibrium. As in the case of symmetric information, the buying decision is only affected by the patience of the investor, that is, all the investors that are patient enough (with $\beta > \hat{\beta}$) buy trees and all the investors who are more impatient do not buy trees. However, the selling decision is now affected also by the type of tree that the investor has. With one price, not only impatient investors sell trees, but also patient investors who have a bad enough tree. These sellers exploit the fact that the price is high relative to the value of their tree. In equilibrium, some investors therefore both buy and sell trees while others, patient investors with a fairly good tree, neither buy not sell but instead just eat their endowment of fruit.

If information is complete, clearly a one price equilibrium distorts the investors' decisions, in the sense that if they could trade for different prices, generically investors would like to deviate. However, we are interested in understanding if a one price equilibrium would be the natural outcome of an environment with private information. From our previous work (Guerrieri and Shimer, 2012), we know that if the trees' quality were the only source of private information, investors would behave differently if they could post different prices with the expectation of being rationed in equilibrium. That is, investors who are impatient enough would choose to sell their trees and, in particular, they would choose to sell better trees at higher prices with a higher probability of being rationed. In this paper, we are

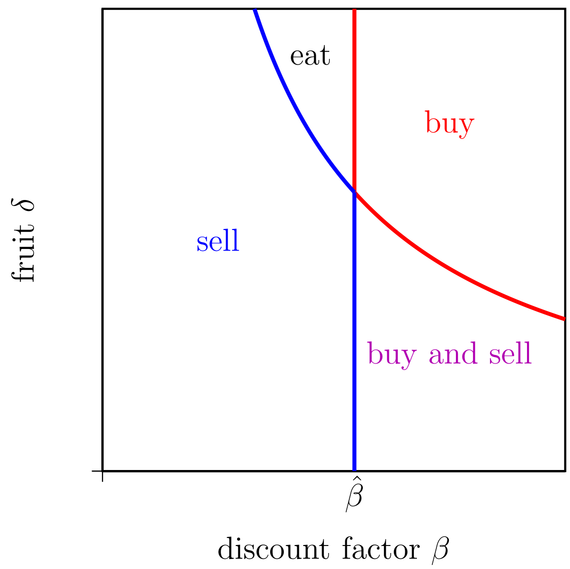

Figure 2: One price equilibrium.

interested in exploring what would happen in an environment where investors can choose the trading price and also their discount factor is private information.

5 Multidimensional Private Information

We now turn to study our main economy in the presence of multidimensional private information. The environment is the same as before, except that we assume that both the tree's type and the degree of impatience of an investor are his own private information. Our equilibrium notion builds on Guerrieri, Shimer and Wright (2010) and Guerrieri and Shimer (2012).

At the beginning of the period, each investor (β, δ, i) knows his discount factor β and the quality of his tree δ. A continuum of markets characterized by a price $p \in R_+$ may open up. Each investor makes an independent buying and selling decision. On the buying side, he has to decide whether to consume his unit of fruit or to use it to buy trees and, if he buys trees, he has to decide at which price. On the selling side, he has to choose whether to sell his tree or not and, if he sells, he has to decide at which price. We assume that each tree or fruit can be brought to only one market, so an effort to sell a tree at a price p is also a commitment not to sell the tree at any other price.

In making their optimal trading decisions, investors must form beliefs about the trading probability and the type trees for sale at any potential price, even those not offered in equilibrium. Let $\Theta(p)$ denote the market tightness associated to price p, that is, the ratio of the amount of fruit buyers want to use to buy at price p, relative to the cost of the trees that sellers want to sell at price p. If $\Theta(p) < 1$, there is not enough fruit to buy all the trees for sale at price p and the sellers are randomly rationed. If instead $\Theta(p) > 1$, there is more fruit than needed to buy all the trees for sale at price p and the buyers are randomly rationed. Specifically, a seller who choose to trade at price p expects to sell with probability $\min\{\Theta(p), 1\}$. Similarly, a buyer who decides to trade at price p expects to buy with probability $\min\{\Theta(p)^{-1}, 1\}$. A seller who is rationed keeps his tree and in period 2 eats the fruit produced by his tree. A buyer who is rationed eats his fruit in period 1.

In addition, let $\Delta(p)$ denote buyers' belief about the average dividend among the trees offered for sale at a price p. If some trees are sold at a price p, these beliefs must be consistent with the quality of trees offered for sale. Otherwise, as long as the buyer-seller ratio $\Theta(p)$ is finite, buyers' belief $\Delta(p)$ must be reasonable in the sense that there must be some set of sellers with average tree quality $\Delta(p)$ who find it weakly optimal to set the price p. This restriction on beliefs restricts the set of possible equilibria by ruling out equilibria that are sustained by weird beliefs about markets that are inactive.

5.1 Equilibrium Definition

We are now ready to define an equilibrium. We let $p_s(\beta, \delta, i)$ denote the optimal sale price for investor (β, δ, i) and $p_b(\beta, \delta, i)$ denote his optimal buy price. We do not offer investors an explicit option not to sell their tree or not to buy a tree, but instead note that in equilibrium, sellers (buyers) can assure that outcome by setting a sufficiently high (low) price.

Definition 3 *An equilibrium is four functions $p_s : \mathbb{B} \times \mathbb{D} \times \mathbb{R}_+ \mapsto \mathbb{R}_+$, $p_b : \mathbb{B} \times \mathbb{D} \times \mathbb{R}_+ \mapsto \mathbb{R}_+$, $\Theta : \mathbb{R}_+ \mapsto [0, \infty]$, $\Delta : \mathbb{R}_+ \mapsto \mathbb{D}$ satisfying the following conditions:*

1. *Optimal Selling Decision: given Θ, for all (β, δ, i)*

$$p_s(\beta, \delta, i) \in \arg\max_{p \geq \beta\delta} \big(\min\{\Theta(p), 1\}(p - \beta\delta)\big);$$

2. *Optimal Buying Decision: given Θ and Δ, for all (β, δ, i)*

$$p_b(\beta, \delta, i) \in \arg\max_{p \geq 0} \left(\min\{\Theta(p)^{-1}, 1\}\left(\frac{\beta\Delta(p)}{p} - 1\right)\right);$$

3. *Beliefs: For all $p \in \mathbb{R}_+$ with $\Theta(p) < \infty$,*

   (a) *if there exists a (β, δ, i) with $p_s(\beta, \delta, i) = p$, $\Delta(p) = \mathbb{E}(\delta | p_s(\beta', \delta', i') = p)$;*

(b) *otherwise there exists a* (β_1, δ_1, i_1) *with* $\delta_1 \le \Delta(p)$, $p \ge \beta_1\delta_1$, *and*

$$\min\{\Theta(p_s(\beta_1, \delta_1, i_1)), 1\}\big(p_s(\beta_1, \delta_1, i_1) - \beta_1\delta_1\big) = \min\{\Theta(p), 1\}\big(p - \beta_1\delta_1\big);$$

and similarly a (β_2, δ_2, i_2) *with* $\delta_2 \ge \Delta(p)$, $p \ge \beta_2\delta_2$, *and*

$$\min\{\Theta(p_s(\beta_2, \delta_2, i_2)), 1\}\big(p_s(\beta_2, \delta_2, i_2) - \beta_2\delta_2\big) = \min\{\Theta(p), 1\}\big(p - \beta_2\delta_2\big);$$

4. *Market Clearing: for all* $p \ge 0$, $d\mu_b(p) = p\,\Theta(p)\,d\mu_s(p)$, *where*

$$\mu_s(p) \equiv \iiint_{p_s(\beta,\delta,i)\le p} di\,d\delta\,d\beta \text{ and } \mu_b(p) \equiv \iiint_{p_b(\beta,\delta,i)\le p} di\,d\delta\,d\beta$$

are the measure of trees for sale at prices below p *and the measure of fruit used to purchase trees at prices below* p. *Moreover, if there exists a* (β, δ, i) *with* $p_s(\beta, \delta, i) = p$ *and* $\Theta(p) > 0$, *then there exists a* (β', δ', i') *with* $p_b(\beta', \delta', i') = p$; *and if there exists a* (β, δ, i) *with* $p_b(\beta, \delta, i) = p$ *and* $\Theta(p) < \infty$, *then there exists a* (β', δ', i') *with* $p_s(\beta', \delta', i') = p$.

The first condition requires that investors make optimal selling decisions. Each seller (β, δ, i) must set an optimal price for her tree.[3] A seller who sets a price p only succeeds in selling with probability $\Theta(p)$ if $\Theta(p) < 1$. In this event, he gets p units of fruit but gives up δ units of fruit tomorrow, which he values at $\beta\delta$. If he fails to sell, he gains nothing. We impose for expositional convenience the restriction that sellers never set a price below their continuation value $\beta\delta$.[4]

The second condition requires that investors make optimal buying decisions. Each buyer (β, δ, i) sets an optimal price for buying trees.[5] A buyer who sets a price p only succeeds in buying with probability $\min\{\Theta(p)^{-1}, 1\}$. In this event, he gives up a unit of fruit and gets $1/p$ trees, each of which he anticipates will produce $\Delta(p)$ fruit next period. If he fails to buy, he gains nothing.

The first part of the third condition imposes that beliefs are consistent with the observed trading patterns whenever possible. If at least one seller sets a price p, then the expected dividend must be the average among the sellers who set that price. The second part of this condition describes beliefs at prices that nobody sets. Intuitively, we require that buyers must be able to rationalize the expected dividend as coming from some probability distribution

[3] There is no loss of generality in assuming that he sells the tree. Attempting to sell at any price $p \ge \beta\delta$ always weakly dominates not selling the tree.

[4] It is never strictly optimal for a seller (β, δ) to set a price $p < \beta\delta$, and is only weakly optimal if $\Theta(p) = 0$ and $\Theta(p') = 0$ for all $p' \ge \beta\delta$.

[5] We prove below that in any equilibrium with trade, $\Theta(p) = 0$ at sufficiently low prices p. Therefore buyers can always be sure to consume their fruit in period 1 by setting a low price.

over sellers, each of whom has a continuation value less than the price and finds setting this price to be weakly optimal. In practice, this means that there must either be some individual with dividend $\Delta(p)$ who finds it optimal to set the price p, or that there must be both an individual with a better tree and an individual with a worse tree, both of whom find this price optimal. In the latter case, appropriate weights on those two individuals justify the expectation $\Delta(p)$.[6]

Finally, the last condition imposes market clearing. It requires that the buyer-seller ratio $\Theta(p)$ at any price p is equal to the ratio of the measure of buyers purchasing at price p to the product of the price and the measure of sellers selling at that price. The last piece of this condition ensures that this holds even if both measures are zero yet a finite number of buyers or sellers sets price p. For notational convenience alone, we do not impose that the buyer-seller ratio is exactly equal to $\Theta(p)$ in this case.

5.2 Parameter Restriction

Let $\Gamma(v) \equiv \mathbb{E}(\delta|\beta\delta = v)$ denote the expected dividend conditional on an individual's continuation value $\beta\delta = v$. It is straightforward to prove that

$$\Gamma(v) \equiv \frac{\int_{\mathbb{D}} g\left(\frac{v}{\delta}, \delta\right) d\delta}{\int_{\mathbb{D}} \frac{1}{\delta} g\left(\frac{v}{\delta}, \delta\right) d\delta},$$

a function of the joint density g, a model primitive We focus our analysis on the case where the following restriction holds:

Assumption 1 *Assume Γ is continuous and increasing.*

We then define some key objects. The first is the lowest continuation value in the economy:

$$\underline{v} = \min_{(\beta,\delta,i)} \beta\delta.$$

The second is the average tree quality among the trees held by investors with the lowest continuation value, $\underline{\gamma} \equiv \Gamma(\underline{v})$. The third is the worst tree quality among those investors with the lowest continuation value:

$$\underline{\delta} = \min_{(\beta,\delta,i)} \delta \text{ s.t. } \beta\delta = \underline{v}.$$

In many cases, $\underline{v}$ will be the worst tree held by any individual, but this need not be the case if tree holdings and discount factors are correlated.

[6]In our previous research (Guerrieri, Shimer and Wright, 2010; Guerrieri and Shimer, 2012), the analogous condition defined a probability distribution over seller types at each price p. None of the results in this paper would change if we used that definition, but the one we use here is simpler to apply.

A distribution function that satisfies our assumption is $G(\beta,\delta)=\beta^{\alpha}\delta^{\alpha+k}$, defined on $\mathbb{B}\times\mathbb{D}=[0,1]^2$, with $\alpha>0$ and $\alpha+k>0$. Then

$$\Gamma(v)=\frac{k\big(1-v^{k+1}\big)}{(1+k)\big(1-v^{k}\big)},$$

which is continuous and increasing. In this case, $\underline{v}=\underline{\delta}=0$ while $\underline{\gamma}=\max\{0,k/(1+k)\}$. We will use this example to illustrate some of our results.

5.3 Equilibrium Characterization

We characterize the set of equilibria through a series of lemmas.

We start with a partial characterization of the equilibrium buyer-seller ratio $\Theta(p)$ by dividing it into three regions. We prove that there is a price $\underline{p}$ such that for all $p<\underline{p}$, $\Theta(p)=\infty$, while for all $p>\underline{p}$, $\Theta(p)<1$; and that there is a price $\bar{p}\geq\underline{p}$ such that for all $p<\bar{p}$, $\Theta(p)>0$ while for all $p>\bar{p}$, $\Theta(p)=0$. We also prove that if $\underline{p}<\bar{p}$, $\Theta(\underline{p})\geq 1$. Note that the proof does not establish that either of the bounds, $\underline{p}$ and $\bar{p}$, is positive or finite, nor does it establish that the bounds differ from each other. In other words, we do not establish that all three regions of the price space exists.

The proof of this lemma uses the fact that every price p with $\Theta(p)<\infty$ must be optimal for some seller, the third part of the definition of equilibrium. Sellers are willing to set a lower price only if they are compensated with a higher trading probability.

Lemma 1 *There are numbers $0\leq\underline{p}\leq\bar{p}\leq\infty$ such that:*

$$\Theta(p)\begin{cases}=\infty & p<\underline{p}\\ \in(0,1) \quad \textit{if} & p\in(\underline{p},\bar{p})\\ =0 & p>\bar{p}.\end{cases}$$

In addition, if $\underline{p}<\bar{p}$ then $\Theta(\underline{p})\geq 1$.

Proof. To find a contradiction, first suppose there are prices $p_1<p_2$ such that $\Theta(p_2)\geq 1$ and $\Theta(p_1)<\infty$. Part 3 of the definition of equilibrium implies that there is a (β,δ,i) with $\beta\delta\leq p_1$ and

$$\min\{\Theta(p_1),1\}(p_1-\beta\delta)\geq\min\{\Theta(p_2),1\}(p_2-\beta\delta).$$

Since $p_1\geq\beta\delta$,

$$p_1-\beta\delta\geq\min\{\Theta(p_1),1\}(p_1-\beta\delta).$$

Since $\Theta(p_2) \geq 1$,

$$\min\{\Theta(p_2), 1\}(p_2 - \beta\delta) = p_2 - \beta\delta.$$

But now combining inequalities implies $p_1 \geq p_2$, a contradiction. Therefore if $\Theta(p_2) \geq 1$, $\Theta(p_1) = \infty$ for all $p_1 < p_2$. We can then define $\underline{p}$ as the maximum p such that $\Theta(p') = \infty$ for all $p' < p$ and conclude that for all $p > \underline{p}$, $\Theta(p) < 1$.

Now take any price p_1 with $\Theta(p_1) = 0$. Again, there is a (β, δ, i) with $\beta\delta \leq p_1$ for whom p_1 is weakly optimal. In particular, for any other price $p_2 > p_1$,

$$\min\{\Theta(p_1), 1\}(p_1 - \beta\delta) \geq \min\{\Theta(p_2), 1\}(p_2 - \beta\delta).$$

Since the left hand side is zero, the right hand side must be as well. Since $p_2 > p_1 \geq \beta\delta$, this proves $\Theta(p_2) = 0$. We can then define $\bar{p}$ as the minimum p such that $\Theta(p') = 0$ for all $p' > p$. Obviously $\bar{p} \geq \underline{p}$. This proves the first part of the result.

Finally, suppose $\bar{p} > \underline{p}$ and $\Theta(\underline{p}) < 1$. Part 3 of the definition of equilibrium implies there is some a (β, δ, i) with $\beta\delta \leq \underline{p}$ who finds the sale price $\underline{p}$ optimal:

$$\min\{\Theta(\underline{p}), 1\}(\underline{p} - \beta\delta) \geq \min\{\Theta(p), 1\}(p - \beta\delta)$$

for all $p \geq \beta\delta$. If $\underline{p} = \beta\delta$, the left hand side is zero, but the right hand is positive at all $p \in (\underline{p}, \bar{p})$. Therefore $\underline{p} > \beta\delta$. But now consider a slightly lower price, $p \in \big(\Theta(\underline{p})\underline{p} + (1 - \Theta(\underline{p}))\beta\delta, \underline{p}\big)$. Since $p < \underline{p}$, $\Theta(p) = \infty$, and so (β, δ, i) prefers selling at price p to selling at $\underline{p}$, a contradiction. This proves $\Theta(\underline{p}) \geq 1$. ■

It is worth stressing that Lemma 1 does not pin down $\Theta(\bar{p})$. We return to the value of the buyer-seller ratio at this price at the end of the proof, in Lemma 8, where we prove that under a mild regularity condition $\Theta(p)$ is continuous at $\bar{p}$ when $\underline{p} < \bar{p}$.

We next prove that the function Θ is decreasing when $p \in [\underline{p}, \bar{p})$ and bound the slope of the function. This implies in particular that $\min\{\Theta(p), 1\}$ is continuous on this set. Our proof again exploits the same property of equilibrium, that every price is optimal for some seller.

Lemma 2 *Take any prices $p_1 < p_2 \leq \bar{p}$ with $\Theta(p_1) < \infty$. Any investor (β_1, δ_1, i_1) who finds the sale price p_1 weakly optimal has $\beta_1\delta_1 < p_1$ and*

$$\frac{-\Theta(p_2)}{p_1 - \beta_1\delta_1} \geq \frac{\Theta(p_2) - \Theta(p_1)}{p_2 - p_1}.$$

In particular, Θ is strictly decreasing on $[\underline{p}, \bar{p}]$. If in addition $p_2 < \bar{p}$, any investor (β_2, δ_2, i_2)

who finds the sale price p_2 weakly optimal has $\beta_2\delta_2 < p_2$ and

$$\frac{\Theta(p_2) - \min\{\Theta(p_1), 1\}}{p_2 - p_1} \geq \frac{-\min\{\Theta(p_1), 1\}}{p_2 - \beta_2\delta_2}.$$

Proof. Since $\Theta(p_1) < \infty$, part 3 of the definition of equilibrium implies there is a (β_1, δ_1, i_1) with $\beta_1\delta_1 \leq p_1$ who finds p_1 weakly optimal:

$$\min\{\Theta(p_1), 1\}\big(p_1 - \beta_1\delta_1\big) \geq \max_{p \geq \beta_1\delta_1} \big(\min\{\Theta(p), 1\}\big(p - \beta_1\delta_1\big)\big).$$

If $p_1 = \beta_1\delta_1$, the left hand side evaluates to 0 and so the right hand side implies $\Theta(p) = 0$ for all $p > p_1$. Lemma 1 then implies $p_1 \geq \bar{p}$, contradicting our assumption that $p_1 < p_2 \leq \bar{p}$ and proving $p_1 > \beta_1\delta_1$. Now the fact that (β_1, δ_1, i_1) prefers p_1 to $p_2 > p_1$ implies

$$\min\{\Theta(p_1), 1\}\big(p_1 - \beta_1\delta_1\big) \geq \Theta(p_2)\big(p_2 - \beta_1\delta_1\big),$$

where we use Lemma 1 to note that $\Theta(p_1) < \infty$ implies $p_1 \geq \underline{p}$, hence $p_2 > \underline{p}$, hence $\Theta(p_2) < 1$. Algebraic manipulation of this inequality proves that

$$\frac{-\Theta(p_2)}{p_1 - \beta_1\delta_1} \geq \frac{\Theta(p_2) - \min\{\Theta(p_1), 1\}}{p_2 - p_1}.$$

Moreover, $\Theta(p_1) \geq \min\{\Theta(p_1), 1\}$, giving the result in the statement of the lemma. Since the left hand side is strictly negative, $\Theta(p_2) < \Theta(p_1)$ and hence $\Theta(p)$ is strictly decreasing.

When $p_2 < \bar{p}$, the statement of the lemma also specifies a lower bound on the slope. We find it symmetrically. Since $p_1 < p_2$ and $\Theta(p_1) < \infty$, Lemma 1 implies $\Theta(p_2) < \infty$ as well. Then the same logic as in the previous paragraph implies that when $p_2 < \bar{p}$, we can find a (β_2, δ_2, i_2) with $\beta_2\delta_2 < p_2$ for whom p_2 is an optimal selling price. Since $p_2 > p_1 \geq \underline{p}$, Lemma 1 implies $\Theta(p_2) < 1$. The fact that (β_2, δ_2, i_2) does not prefer price p_1 to p_2 implies either that $p_1 \geq \beta_2\delta_2$ and

$$\Theta(p_2)\big(p_2 - \beta_2\delta_2\big) \geq \min\{\Theta(p_1), 1\}\big(p_1 - \beta_2\delta_2\big),$$

or that $p_1 < \beta_2\delta_2$, in which case the inequality holds trivially. The result then follows from algebraic manipulation of this inequality. ■

The next lemma shows that an investor with a higher continuation value for his tree attempts to sell it at a weakly higher price. The proof is simply based on the preferences of the seller, i.e. part 1 of the definition of equilibrium.

Lemma 3 *Take any* (β_1, δ_1, i_1) *and* (β_2, δ_2, i_2) *with* $\beta_1\delta_1 < \beta_2\delta_2$ *and either* $\Theta(p_s(\beta_1, \delta_1, i_1)) > 0$ *or* $\Theta(p_s(\beta_2, \delta_2, i_2)) > 0$. *Then* $p_s(\beta_2, \delta_2, i_2) \geq p_s(\beta_1, \delta_1, i_1) \geq \underline{p}$.

Proof. To start, suppose there is a (β, δ, i) with $p_s(\beta, \delta, i) \equiv p < \underline{p}$. Part 1 of the definition of equilibrium implies $p \geq \beta\delta$ and $\min\{\Theta(p), 1\}(p - \beta\delta) \geq \min\{\Theta(p'), 1\}(p' - \beta\delta)$, where $p' = \frac{1}{2}(p + \underline{p})$. Lemma 1 implies both minima evaluate to 1, and so this implies $p \geq p'$, which contradicts $p < \underline{p}$. Therefore $p_s(\beta, \delta, i) \geq \underline{p}$.

For the remainder of this proof, let $p_1 \equiv p_s(\beta_1, \delta_1, i_1) \geq \underline{p}$ and $p_2 \equiv p_s(\beta_2, \delta_2, i_2) \geq \underline{p}$. If $p_2 \geq \beta_1\delta_1$, part 1 of the definition of equilibrium implies

$$\min\{\Theta(p_1), 1\}(p_1 - \beta_1\delta_1) \geq \min\{\Theta(p_2), 1\}(p_2 - \beta_1\delta_1). \tag{4}$$

If $p_2 < \beta_1\delta_1$, the right hand side of this inequality is non positive and so the inequality also obtains. The same logic implies

$$\min\{\Theta(p_2), 1\}(p_2 - \beta_2\delta_2) \geq \min\{\Theta(p_1), 1\}(p_1 - \beta_2\delta_2). \tag{5}$$

Now to find a contradiction, suppose $p_1 > p_2 \geq \underline{p}$. We now consider two cases:

1. $\bar{p} \geq p_1 > \underline{p}$. Lemma 1 implies $\Theta(p_1) < 1$ and Lemma 2 implies $\Theta(p_1) < \Theta(p_2)$. Adding inequalities (4)–(5), we obtain

   $$(\Theta(p_1) - \min\{\Theta(p_2), 1\})(\beta_2\delta_2 - \beta_1\delta_1) \geq 0$$

   Since $\beta_2\delta_2 > \beta_1\delta_1$, this implies $\Theta(p_1) \geq \min\{\Theta(p_2), 1\}$, a contradiction.

2. $p_1 > \bar{p}$. Lemma 1 implies $\Theta(p_1) = 0$ and so $\Theta(p_2) > 0$ by assumption. Part 1 of the definition of equilibrium implies $p_2 \geq \beta_2\delta_2$, while $\beta_2\delta_2 > \beta_1\delta_1$ by assumption. This implies the right hand side of inequality (4) is strictly positive, while the left hand side is zero, a contradiction. ■

We next prove the result in Lemma 3 is strict: that any two investors selling at the same price must have the same continuation value, or equivalently, an investor with a higher continuation value for his tree attempts to sell it at a higher price. This result uses much more of the structure of equilibrium, and in particular uses Assumption 1. We prove that if sellers with two distinct continuation values find the same price optimal, then a buyer would prefer to purchase trees at a slightly higher price, knowing that the expected dividend would be discretely higher.

Lemma 4 *Impose Assumption 1. Take any* (β_1, δ_1, i_1) *and* (β_2, δ_2, i_2) *with* $\beta_1\delta_1 < \beta_2\delta_2$ *and either* $\Theta(p_s(\beta_1, \delta_1, i_1)) > 0$ *or* $\Theta(p_s(\beta_2, \delta_2, i_2)) > 0$*. Then* $p_s(\beta_2, \delta_2, i_2) > p_s(\beta_1, \delta_1, i_1) \geq \underline{p}$*.*

Proof. We prove that if $p_s(\beta_1, \delta_1, i_1) = p_s(\beta_2, \delta_2, i_2) = p$ and $\Theta(p) > 0$, $\beta_1\delta_1 = \beta_2\delta_2$. Together with Lemma 3, this establishes our result. Let $v_2 = \sup\{\beta\delta | p_s(\beta, \delta, i) = p\}$ and $v_1 = \inf\{\beta\delta | p_s(\beta, \delta, i) = p\}$. To find a contradiction, assume $v_2 > v_1$. Note that part 1 of the definition of equilibrium imposes $p \geq v_2$, while Lemma 3 implies $p \geq \underline{p}$.

We first prove that for any (β, δ, i) with $v_2 > \beta\delta > v_1$, $p_s(\beta, \delta, i) = p$. Since $\beta\delta > v_1$ and $\Theta(p) > 0$, Lemma 3 implies $p_s(\beta, \delta, i) \geq p$. Since $\beta\delta < v_2$, the same lemma implies $p_s(\beta, \delta, i) \leq p$. Therefore $p_s(\beta, \delta) = p$.

Since $\Gamma(v)$ is increasing by Assumption 1, $\Gamma(v_2) > \Gamma(v_1)$. Moreover, since all sellers (β, δ) with $\beta\delta \in (v_1, v_2)$ set price p and the support of the type distribution is convex, $\Delta(p) \in (\Gamma(v_1), \Gamma(v_2))$. In addition, for $p' > p$ and (β', δ', i') with $p_s(\beta', \delta', i') = p'$, Lemma 3 implies $\beta'\delta' \geq v_2$ so $\Delta(p') \geq \Gamma(v_2)$. In particular, for all $p' \in \big(p, p\Gamma(v_2)/\Delta(p)\big)$, $\Delta(p')/p' > \Delta(p)/p$.

Next, we use part 4 of the definition of equilibrium, market clearing, evaluated at p. Since $\Theta(p) > 0$ and a positive measure $d\mu_s(p)$ of sellers set price p, all (β, δ, i) with $\beta\delta \in (v_1, v_2)$, it must the case that there is also a positive measure of buyers, $d\mu_b(p) = p\Theta(p)d\mu_s(p)$. Since $d\mu_b(p) \leq 1$, the total measure of potential buyers, $\Theta(p) < \infty$ whenever a positive measure of sellers sets the price p. Then part 2 of the definition of equilibrium implies that for any (β, δ, i) with $p_b(\beta, \delta, i) = p$, $\beta\Delta(p) \geq p$, since it is always possible to set a price $p' < \underline{p}$ with $\Theta(p') = \infty$. The same condition also implies that for all p',

$$\min\{\Theta(p)^{-1}, 1\}\left(\frac{\beta\Delta(p)}{p} - 1\right) \geq \min\{\Theta(p')^{-1}, 1\}\left(\frac{\beta\Delta(p')}{p'} - 1\right).$$

Recall that at any $p' \in \big(p, p\Gamma(v_2)/\Delta(p)\big)$, $\Delta(p)/p < \Delta(p')/p'$, so $\beta\Delta(p') > p'$. If $\beta\Delta(p) = p$, it follows that $\Theta(p') = \infty$. If $\beta\Delta(p) > p$, we still must have $\min\{\Theta(p)^{-1}, 1\} > \min\{\Theta(p')^{-1}, 1\}$, so in particular $\Theta(p') > 1$. However Lemma 1 implies $\Theta(p') < 1$ for all $p' > \underline{p}$, a contradiction of $p' > p \geq \underline{p}$. ■

We next take advantage of the fact that each price is set by only one continuation value to find an expression for the buyer-seller ratio only as a function of the continuation value of the seller who sets that price.

Lemma 5 *Impose Assumption 1 and assume* $\bar{p} > \underline{p}$*. Define* $V(p)$ *such that* $V(p) = \beta\delta$

implies p is a weakly optimal sale price for every (β, δ, i). Then for all $p \in (\underline{p}, \bar{p})$,

$$\Theta(p) = \exp\left(-\int_{\underline{p}}^{p} \frac{1}{p' - V(p')} dp'\right). \tag{6}$$

Proof. Lemma 4 implies that it is possible to define a nondecreasing, continuous function $V(p)$ as described in the statement of the lemma. Then Lemma 2 implies that for any $p < p'$ in the interval $(\underline{p}, \bar{p})$,

$$\frac{-\Theta(p')}{p - V(p)} \geq \frac{\Theta(p') - \Theta(p)}{p' - p} \geq \frac{-\Theta(p)}{p' - V(p')}.$$

Taking the limit as $p' \to p$, we get that both of the extreme limits converge and so

$$\Theta'(p) = \frac{-\Theta(p)}{p - V(p)}.$$

It is straightforward to solve this differential equation to get

$$\Theta(p) = \theta_0 \exp\left(-\int_{\underline{p}}^{p} \frac{1}{p' - V(p')} dp'\right)$$

for some constant θ_0. Since $\Theta(p) < 1$ for all $p > \underline{p}$, $\theta_0 \leq 1$. Since $\Theta(p) \geq 0$ for all p, $\theta_0 \geq 0$. And if $\theta_0 = 0$, Lemma 1 implies $\bar{p} = \underline{p}$. This proves $\theta_0 \in (0, 1]$.

Now to find a contradiction, suppose $\theta_0 < 1$. For all $p \in [\underline{p}, \bar{p})$ and any (β, δ) with $\beta\delta = V(p)$, Part 1 of the definition of equilibrium implies $\beta\delta < p$; otherwise any price $p' \in (p, \bar{p})$ would be preferred by the seller. In particular, continuity of V and $V(\underline{p}) < \underline{p}$ implies that there exists $p > \underline{p}$ such that $\theta_0 p + (1 - \theta_0)V(p) < \underline{p}$ and so $V(p) < \underline{p}$. Now for any (β, δ, i) with $\beta\delta = V(p)$, setting the price $\underline{p}$ and selling for sure dominates setting the price p and selling with probability $\Theta(p) < \theta_0$, contradicting part 1 of the definition of equilibrium. This proves $\theta_0 = 1$ and establishes equation (6). ■

We next show that any investor who sells at the price $\bar{p}$ is just indifferent about selling. The details of the proof depend on whether $\Theta(p) > 0$.

Lemma 6 *Impose Assumption 1 and assume $\bar{p} > \underline{p}$. Let (β, δ, i) with $\beta\delta \leq \bar{p}$ be a seller who finds price $\bar{p}$ weakly optimal. Then $\beta\delta = \bar{p}$.*

Proof. To find a contradiction, suppose $\bar{p} > \beta\delta$. We consider two cases. First, if $\Theta(\bar{p}) = 0$, then (β, δ, i) earns higher profits selling at any price $p \in (\beta\delta, \bar{p})$ than at $\bar{p}$, since $\Theta(p)(p-\beta\delta) >$

$0 = \Theta(\bar{p})(\bar{p} - \beta\delta)$. Second, if $\Theta(\bar{p}) > 0$, take any (β', δ', i') with $\beta'\delta' \in (\beta\delta, \bar{p})$. From Lemmas 3 and 4, we know that $p_s(\beta', \delta', i') > \bar{p}$ because $\beta'\delta' > \beta\delta$ and hence, from Lemma 1, we know that $\Theta(p_s(\beta', \delta', i')) = 0$. Then, such a seller would prefer to sell for $\bar{p}$, since $\Theta(\bar{p})(\bar{p} - \beta'\delta') > 0 = \Theta(p_s(\beta', \delta', i'))(p_s(\beta', \delta', i') - \beta'\delta')$. This is a contradiction, which proves $\bar{p} = \beta\delta$. ■

We use this to establish that every seller (β, δ, i) with $\bar{p} \geq \beta\delta > \underline{v}$ finds a unique price $p \in (\underline{p}, \bar{p})$ optimal, where $\underline{v}$ is the minimum value of $\beta\delta$ in the population.

Lemma 7 *Impose Assumption 1 and assume $\bar{p} > \underline{p}$. Take any investors (β, δ, i) and (β', δ', i') with $\bar{p} > \beta\delta = \beta'\delta' > \underline{v}$. Then $p_s(\beta, \delta, i) = p_s(\beta', \delta', i') \in (\underline{p}, \bar{p})$.*

Proof. To find a contradiction, assume $p \equiv p_s(\beta, \delta, i) < p_s(\beta', \delta', i') \equiv p'$. Since $\underline{v} < \beta\delta$, $p > \underline{p}$ by Lemma 4, while $\bar{p} > \beta'\delta'$ implies $p' < \bar{p}$ by Lemma 6. Since Γ is continuous and increasing by Assumption 1, there exists an $\varepsilon < \min\{\beta\delta - \underline{v}, \bar{p} - \beta\delta\}$ such that if both $v_1 \in (\underline{v}, \beta\delta)$ and $v_2 \in (\beta\delta, \bar{p})$ lie within an ε-ball of $\beta\delta$, $1 < \Gamma(v_2)/\Gamma(v_1) < p'/p$. Our proof establishes that there cannot be buyers for all the trees sold by sellers with continuation value v_2.

First fix p_1 such that there exists a (β_1, δ_1, i_1) with $\beta_1\delta_1 = v_1$, $p_s(\beta_1, \delta_1, i_1) = p_1$, and $\Delta(p_1) \geq \Gamma(v_1)$. Such a price must exist since all investors with continuation value v_1 attempt to sell their trees at a different price than trees held by any investor with a different continuation value; and the average dividend of those trees is $\Gamma(v_1)$. Since $\beta_1\delta_1 = v_1 \in (\underline{v}, \beta\delta)$, $p_1 \in (\underline{p}, p)$ by Lemma 4.

Similarly, fix p_2 such that there exists a (β_2, δ_2, i_2) with $\beta_2\delta_2 = v_2$, $p_s(\beta_2, \delta_2, i_2) = p_2$, and $\Delta(p_2) \leq \Gamma(v_2)$. The logic of why such a price exists is symmetric. Since $\beta_2\delta_2 = v_2 \in (\beta'\delta', \bar{p})$, $p_2 \in (p', \bar{p})$ by Lemmas 4 and 6.

Finally, part 4 of the definition of equilibrium implies that there must be some $(\tilde{\beta}, \tilde{\delta}, \tilde{i})$ with $p_b(\tilde{\beta}, \tilde{\delta}, \tilde{i}) = p_2$. But

$$\frac{\Delta(p_2)}{p_2} \leq \frac{\Gamma(v_2)}{p'} < \frac{\Gamma(v_1)}{p} \leq \frac{\Delta(p_1)}{p_1}.$$

The first inequality uses $\Delta(p_2) \leq \Gamma(v_2)$ and $p_2 > p'$. The second uses $\Gamma(v_2)/\Gamma(v_1) < p'/p$. The third uses $\Gamma(v_1) \leq \Delta(p_1)$ and $p_1 \leq p$. Lemma 1 implies $\Theta(p_1) < \Theta(p_2) < 1$ and so if $\tilde{\beta} > 0$,

$$\min\{\Theta(p_2)^{-1}, 1\} \left(\frac{\tilde{\beta}\Delta(p_2)}{p_2} - 1 \right) < \min\{\Theta(p_1)^{-1}, 1\} \left(\frac{\tilde{\beta}\Delta(p_1)}{p_1} - 1 \right),$$

which contradicts $p_b(\tilde{\beta}, \tilde{\delta}, \tilde{i}) = p_2$. If $\tilde{\beta} = 0$,

$$\min\{\Theta(p_2)^{-1}, 1\} \left(\frac{\tilde{\beta}\Delta(p_2)}{p_2} - 1 \right) < 0 = \min\{\Theta(\tilde{p})^{-1}, 1\} \left(\frac{\tilde{\beta}\Delta(\tilde{p})}{\tilde{p}} - 1 \right)$$

for any $\tilde{p} < \underline{p}$ by Lemma 1, again contradicting $p_b(\tilde{\beta}, \tilde{\delta}, \tilde{i}) = p_2$. ■

Lemma 7 does not claim that investors with the lowest continuation value $\underline{v}$ have a unique optimal sale price. Indeed, this is not generally true and is the source of our multiple equilibria.

5.4 A Continuum of Equilibria

We now establish our main theoretical result, that there is a continuum of equilibrium payoffs whenever the worst tree held by an investor with the lowest continuation value is worse than the average tree held by such an investor, $\underline{\delta} < \underline{\gamma}$. If the two are equal, then the equilibrium is unique.

Proposition 3 *Impose Assumption 1. Fix $\hat{\beta} \in \mathbb{B}$. Define*

$$\hat{\theta} \equiv \frac{\int_{\mathbb{D}} \int_{\hat{\beta}}^{\infty} g(\beta, \delta)\, d\beta\, d\delta}{\int_{\underline{v}}^{\bar{p}} \hat{\beta}\Gamma(v) \exp\left(- \int_{\underline{v}}^{v} \frac{\hat{\beta}\Gamma'(v')}{\hat{\beta}\Gamma(v') - v'} dv' \right) h(v) dv}, \tag{7}$$

where $h(v) \equiv \int_{\mathbb{D}} \frac{1}{\delta} g(v/\delta, \delta)\, d\delta$ is the density of continuation values. If $1 \geq \hat{\theta} \geq \frac{\hat{\beta}\underline{\delta} - \underline{v}}{\hat{\beta}\underline{\gamma} - \underline{v}}$, then there exists an equilibrium characterized by three thresholds $\underline{p} \leq \hat{p} < \bar{p}$ satisfying:

- $\underline{p} \equiv \hat{\theta}\hat{\beta}\underline{\gamma} + (1 - \hat{\theta})\underline{v}$
- $\hat{p} \equiv \hat{\beta}\underline{\gamma}$
- *$\bar{p}$ is the smallest solution to $\bar{p} = \hat{\beta}\Gamma(\bar{p})$*

In any such equilibrium,

- *Any investor with $\beta\delta = \underline{v}$ is indifferent about selling his tree at any price $p \in [\underline{p}, \hat{p}]$. At these prices, the sale probability is $\Theta(p) = \hat{\theta}(\hat{p} - \underline{v})/(p - \underline{v})$.*
- *Any investor with $\beta\delta = v \in (\underline{v}, \bar{p})$ sells his tree at the price $P(v) = \hat{\beta}\Gamma(v)$. In equilibrium he sells with probability*

$$\Theta(P(v)) = \hat{\theta} \exp\left(- \int_{\underline{v}}^{v} \frac{\hat{\beta}\Gamma'(v')}{\hat{\beta}\Gamma(v') - v'} dv' \right).$$

- *Any investor with $\beta\delta = v \geq \bar{p}$ is indifferent about selling his tree at any price $p \geq v$. In equilibrium he sells with probability $\Theta(p) = 0$, except possibly $\Theta(\bar{p}) > 0$.*

- *Any investor with $\beta > \hat{\beta}$ is indifferent about buying trees at any price $p \in [\hat{p}, \bar{p})$. Any investor with $\beta < \hat{\beta}$ buys with probability 0 at a price $p < \underline{p}$.*

Conversely, any equilibrium with $\bar{p} > \underline{p}$ must satisfy each of these conditions.

Proof. To construct an equilibrium, we must first define the selling prices for all investors. Assume $p_s(\beta, \delta, i) = P(\beta\delta)$ where $P(v) \equiv \max\{v, \hat{\beta}\Gamma(v)\}$. It is straightforward to verify that these sale prices are weakly optimal for all sellers given the function $\Theta(p)$. Note in particular that the expression for $\Theta(P(v))$ is obtained from equation (6), first using $V(p') = \underline{v}$ when $p' \in [\underline{p}, \hat{p}]$, then using integration by substitution with $p' = P(v')$ when $p' \in (\hat{p}, \bar{p})$.

Turn next to the belief function $\Delta(p)$ and buyers' behavior. The third equilibrium condition implies $\Delta(P(v)) = \Gamma(v)$ for all $v \in [\underline{v}, \bar{p}]$, and so all prices $p \in [\underline{p}, \bar{p}]$ have the same price-dividend ratio, $p/\Delta(p) = \hat{\beta}$. At lower prices, $\Delta(p) = \underline{\delta}$ is consistent with the third equilibrium condition. Since $\hat{\theta} \geq (\hat{\beta}\underline{\delta} - \underline{v})/(\hat{\beta}\underline{\gamma} - \underline{v})$, the price dividend ratio is lower when $p \in [\underline{p}, \hat{p})$, $p/\underline{\delta} \leq \hat{\beta}$, and so buying at prices in this interval is not optimal. At still lower prices, $\Theta(p) = \infty$ and so buying at these prices is not feasible. On the other hand, at higher prices $p > \bar{p}$ that someone charges, the price-dividend ratio exceeds $\hat{\beta}$ by construction. And at higher prices that no one charges, assume $\Delta(p) = \Gamma(\bar{p})$ and so the price-dividend ratio is again higher than $\hat{\beta}$.

The final equilibrium condition is the fruit market clearing condition, which we can write as

$$\int_{\mathbb{D}} \int_{\hat{\beta}}^{\infty} g(\beta, \delta)\, d\beta\, d\delta = \int_{\underline{v}}^{\hat{p}} P(v)\Theta(P(v))h(v)dv.$$

The functional forms allow us to solve this for $\hat{\theta}$, so equation (7) implies that the fruit market clears. By allocating the fruit of investors with $\beta > \hat{\beta}$ appropriately across markets, we can then get all other markets to clear at the appropriate buyer-seller ratio. This proves that the conditions in the Lemma characterize an equilibrium.

The proof that an equilibrium must satisfy these conditions follows from the preceding lemmas. Lemmas 4, 6 and 7 imply that every investor (β, δ, i) with $\beta\delta \in (\underline{v}, \bar{p})$ has a unique optimal sale price $p_s(\beta, \delta, i) \in (\underline{p}, \bar{p})$, continuously increasing in $\beta\delta$. Lemma 1 implies that $\Theta(p_s(\beta, \delta, i)) < 1$. Hence, optimal buying decisions require that, for some investor to be willing to purchase all these goods, $\Gamma(\beta\delta)/p_s(\beta, \delta, i)$ is same for all such (β, δ, i), say $p_s(\beta, \delta, i) = \hat{\beta}\Gamma(\beta\delta)$ for some $\hat{\beta}$. In particular, optimal buying decisions imply that any investor (β', δ', i') with $\beta' > \hat{\beta}$ is willing to purchase any of these goods, while if $\beta' < \hat{\beta}$, the investor prefers to consume her fruit in the first period.

Lemma 4 implies that any investor with continuation value $\underline{v}$ must choose a lower sale price than any investor with a higher continuation value, but Lemma 7 does not imply that the choice is unique. We therefore let $[\underline{p}, \hat{p}]$ denote the range of optimal sale prices for investors with the lowest continuation value. It follows that $\hat{p} = \hat{\beta}\underline{\gamma}$, while investors with higher continuation values set higher prices.

We turn next to the buyer-seller ratio. By the construction of $\hat{p}$, only sellers with $\beta\delta = \underline{v}$ find it optimal to sell at any price $p \in (\underline{p}, \hat{p}]$. Hence, Lemma 5 implies that for any p in this range, $\Theta(p) = (\underline{p} - \underline{v})/(p - \underline{v})$. For $p \in (\hat{p}, \bar{p})$, there is a unique continuation value $V(p) = \Gamma^{-1}(p/\hat{\beta})$ that finds this price optimal. Again, Lemma 5 determines the functional form for Θ over this range.

Finally, we pin down the thresholds. First, since a seller with $\beta\delta = \bar{p}$ finds $\bar{p}$ optimal by Lemma 6, $\bar{p} = \hat{\beta}\Gamma(\bar{p})$. Second, we have already proved that $\hat{p} = \hat{\beta}\underline{\gamma}$. The seller with the lowest continuation value may set a price weakly lower than this, $\underline{p} \leq \hat{p}$. Since her dividend is at least equal to $\underline{\delta}$, buyers' optimality implies $\underline{p} \geq \hat{\beta}\underline{\delta}$ as well. ■

An immediate implication of this Lemma is that if $\underline{\gamma} = \underline{\delta}$, $\hat{\beta}$ is uniquely determined in equilibrium by the requirement that $\hat{\theta} = 1$. This then pins down equilibrium trading patterns for each type of investor. On the other hand, if $\underline{\gamma} > \underline{\delta}$, a range of different $\hat{\beta}$ are consistent with equilibrium. In equilibria in which $\hat{\beta}$ is larger, every investor sets a higher sale price (except those who sell with zero probability) and some investors stop buying trees. This has a real consequences for equilibrium payoffs.

If $\underline{\delta} = 0$, $\underline{v} = 0$ as well and so one equilibrium has $\hat{\theta} = 0$ and $\hat{\beta}$ equal to the population maximum.[7] There is no trade. On the other hand, so long as $\underline{\gamma} > 0$, there exists an equilibrium with $0 < \underline{p} \leq \hat{p} < \bar{p}$ and $\Theta(p) \in (0, 1)$ for all $p \in (\underline{p}, \bar{p})$. If $\underline{\delta} < \underline{\gamma}$, there is a continuum of such equilibria.

Figure 3 illustrates investors' behavior in one particular equilibrium. Qualitatively, the equilibrium looks similar to the one price equilibrium depicted in Figure 2. In particular, investors are divided into four groups. Patient investors with a high quality tree buy other trees. Impatient investors with a low quality tree sell their tree. There are also patient investors with a low quality tree who sell their tree and buy other trees; and somewhat impatient investors with a high quality tree who neither buy nor sell trees but simply eat their fruit.

Despite these superficial similarities, the one price equilibrium is quite different than our

[7] There is a technical issue with our notation in this case, since $\underline{p} = 0 < \bar{p}$ but $\Theta(p) = 0$ for all $p > 0$. Lemma 1 would define $\bar{p} = 0$ as well. Nevertheless, this is a valid equilibrium.

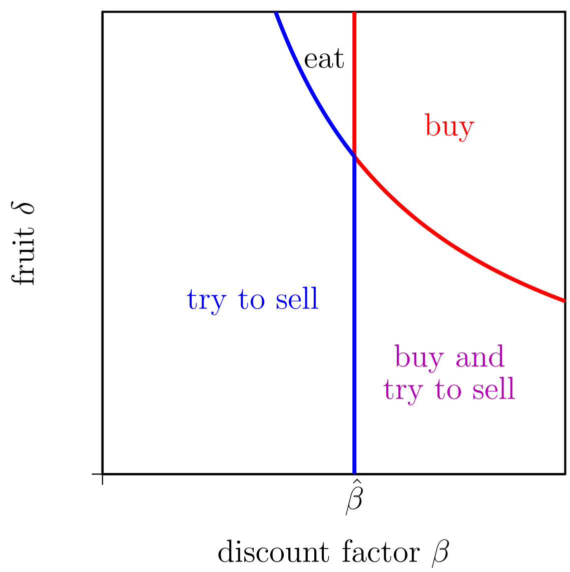

Figure 3: Behavior in partial pooling equilibrium.

equilibrium. In our equilibrium, many investors attempt to sell their tree but do not succeed. Indeed, investors with a continuation value v below $\bar{p}$ choose to hold out for a high price $\hat{\beta}\Gamma(v) > v$ and sell with a low probability, when they could attempt to sell at a lower price that still exceeds their continuation value and have a higher probability of success.

Another important difference is that generically there are a finite number of one price equilibria. In contrast, our model has multiple equilibria if $\underline{\delta} < \underline{\gamma}$. We turn next to a worked out example to illustrate the nature of our multiple equilibria.

5.5 An Example

Assume $G(\beta, \delta) = \beta\delta^2$ so $\underline{v} = 0$ and $\Gamma(v) = \frac{1+v}{2}$. In this case, $\underline{p} \in [0, \frac{1}{2}\hat{\beta}]$, $\hat{p} = \frac{1}{2}\hat{\beta}$, and $\bar{p} = \hat{\beta}/(2-\hat{\beta})$. We can solve explicitly for the buyer-seller ratio:

$$\Theta(p) = \begin{cases} \infty & \text{if } p \in [0, \underline{p}) \\ \underline{p}/p & \text{if } p \in \left(\underline{p}, \hat{p}\right) \\ (\underline{p}/\hat{p})\left(2\hat{\beta}^{-2}(\hat{\beta} - (2-\hat{\beta})p)\right)^{\frac{\hat{\beta}}{2-\hat{\beta}}} & \text{if } p \in [\hat{p}, \bar{p}) \\ 0 & \text{if } p \in [\bar{p}, \infty), \end{cases}$$

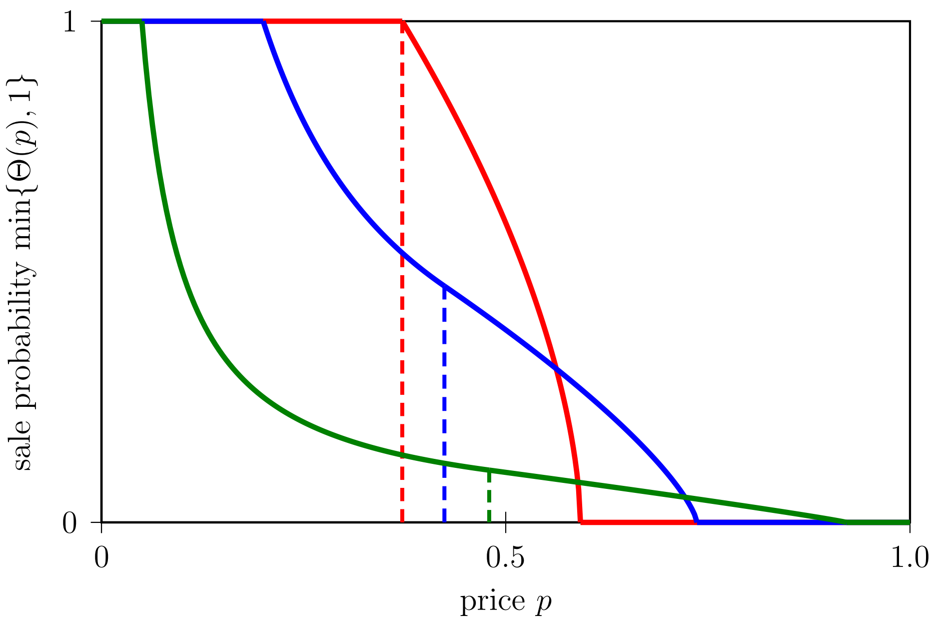

Figure 4: This illustrates three different equilibrium buyer-seller ratios with $G(\beta, \delta) = \beta\delta^2$. The red line corresponds to the case of $\underline{p} = \hat{p} = 0.3720$, which implies $\hat{\beta} = 0.7441$. The blue line has $\underline{p} = 0.2$, which implies $\hat{\beta} = 0.8483$. The green line has $\underline{p} = 0.05$ and so $\hat{\beta} = 0.9592$. The dashed lines indicate the value of $\hat{p}$ in each equilibrium.

where we take advantage of the closed form to solve the integral explicitly. The market clearing condition then implies, after a mess of algebra, that

$$1 - \hat{\beta} = \underline{p}\frac{\hat{\beta}(12 - 7\hat{\beta})}{12 - 7\hat{\beta} + \hat{\beta}^2}.$$

It is easy to show that $\hat{\beta}$ falls monotonically from 1 when $\underline{p} = \hat{\theta} = 0$ to 0.7441 when $\underline{p} = \hat{p} = 0.3720$ and $\hat{\theta} = 1$. Any value of $\hat{\theta}$ (or $\underline{p}$) in this range corresponds to an equilibrium.

A curious feature of these equilibria is that the probability of selling is higher at high prices but lower at low prices in "less liquid" equilibria with lower θ. Indeed, the trading probability at the lower bound $\hat{p} = \frac{1}{2}\hat{\beta}$ is unambiguously lower with $\hat{\beta}$ is larger (since $\underline{p}$ is lower), while the upper bound $\frac{\hat{\beta}}{2-\hat{\beta}}$ is unambiguously increasing and so the trading probability rises at those prices. We illustrate this in Figure 4 by indicating the function $\Theta(p)$ in three different equilibria, all consistent with the parameterization $G(\beta, \delta) = \beta\delta^2$.

It is worth stressing that for any value of $\underline{p}$ consistent with equilibrium, we can construct an equilibrium in which some seller offers each price $p \in [\underline{p}, 1]$. Since $\beta\delta \in [0, 1]$ in this example, higher prices are never paid and so are uninteresting, while lower prices never attract any sellers are so beliefs are necessarily arbitrary.

To prove this, first assume that any seller (β, δ, i) with $\beta\delta > \bar{p}$ sets a price $p_s(\beta, \delta, i) = \beta\delta$, which is weakly optimal. Second, Proposition 3 implies that any price $p \in (\hat{p}, \bar{p}]$ is offered by a seller with continuation value $\Gamma^{-1}(p/\hat{\beta})$. Third, assume $p_s(0, p/\hat{\beta}, 0) = p$ for all $p \in (\underline{p}, \hat{p})$. This seller finds that price weakly optimal; and if he alone offers the price, a buyer with $\beta > \hat{\beta}$ is willing to purchase at that price. Finally, assume that $p_s(\beta, \delta, i) = \hat{p}$ for all other sellers (β, δ, i) with $\beta\delta = 0$. This is an equilibrium.[8]

5.6 A Detail

The proposition does not determine $\Theta(\bar{p})$. Under a slight additional restriction, we can prove that $\Theta(\bar{p}) = 0$:

Lemma 8 *Impose Assumption 1 and assume $\bar{p} > \underline{p}$. Assume that for all $\bar{v}$, there exists a $\gamma > 0$ such that for all $v < \bar{v}$, $\Gamma(v) \leq \Gamma(\bar{v}) + \gamma(v - \bar{v})$. Then in any equilibrium, $\Theta(\bar{p}) = 0$.*

Proof. For $p \in (\underline{p}, \bar{p})$, we have

$$-\log(\Theta(p)) = \int_{\underline{p}}^{p} \frac{1}{p' - \Gamma^{-1}(p'/\hat{\beta})} dp'$$

with $p' > \Gamma^{-1}(p'/\hat{\beta})$ for $p' \in (\underline{p}, \bar{p})$ and $\bar{p} = \Gamma^{-1}(\bar{p}/\hat{\beta})$. We are interested in proving that the right hand side converges to infinity when p converges to $\bar{p}$. Now fix $\gamma \in (0, 1/\hat{\beta})$ such that for all $v < \bar{p}$, $\Gamma(v) \leq \Gamma(\bar{p}) + \gamma(v - \bar{p})$. Equivalently, let $p' = \hat{\beta}\Gamma(v)$ and note that $\bar{p} = \hat{\beta}\Gamma(\bar{p})$. Then for all $p' \in (\underline{p}, \bar{p})$, $\Gamma^{-1}(p'/\hat{\beta}) \geq \bar{p} + (p' - \bar{p})/(\hat{\beta}\gamma)$. In particular,

$$\int_{\underline{p}}^{\bar{p}} \frac{1}{p' - \Gamma^{-1}(p'/\hat{\beta})} dp' \geq \frac{\hat{\beta}\gamma}{1 - \hat{\beta}\gamma} \int_{\underline{p}}^{\bar{p}} \frac{1}{\bar{p} - p'} dp'.$$

Since we fixed $\gamma \in (0, 1/\hat{\beta})$, $\frac{\hat{\beta}\gamma}{1-\hat{\beta}\gamma} > 0$, while the integral is infinite. The original integral is therefore unbounded as well, proving the result. ■

It is possible to construct an example showing that strict monotonicity of Γ is not sufficient for this result. The key is to construct an example in which $\Gamma'(\bar{p}) = 0$. Assume $\Gamma(v) = \frac{1}{3}\big(1 + 2\sqrt{v(1-v)}\big)$ for $v \in [0, 1/2]$ and $\Gamma(v) = 1 - \frac{4}{3}v(1-v)$ for $v \in (1/2, 1]$. Also assume that $\hat{\beta} = 3/4$. All of this could be made precise through some careful choice of $G(\beta, \delta)$.

[8] This equilibrium assumed that almost all of the sellers with $\beta\delta = \underline{v}$ set price $\hat{p}$. We can also construct an equilibrium in which many of them do, say $p_s(0, p/\hat{\beta}, i) = p$ for all $p \in (\underline{p}, \hat{p})$ and $p_s(\beta, \delta, i) = \underline{p}$ for all others with $\beta\delta = 0$. This raises $\Delta(\hat{p})$, the quality of the assets for sale at $\underline{p}$. But by setting $\Theta(\underline{p}) = \infty$, we can ensure that this only attracts buyers with a discount factor below $\hat{\beta}$.

Then it is possible to show that $\underline{p} = \hat{\beta}\Gamma(0) = 1/4$ and $\bar{p} = \hat{\beta}\Gamma(\bar{p}) = 1/2$. Some simple algebra gives $\Gamma^{-1}(p) = \frac{1}{2}\big(1 - \sqrt{3p(2-3p)}\big)$ for $p \in [1/3, 2/3]$ and $\Gamma^{-1}(p) = \frac{1}{2}\big(1 + \sqrt{3p-2}\big)$ for $p \in (2/3, 1]$. Now solve the integral

$$\int_{\underline{p}}^{\bar{p}} \frac{1}{p' - \Gamma^{-1}(p'/\hat{\beta})} dp' = \frac{1}{5}(\pi + \log 8),$$

so $\lim_{p \nearrow \bar{p}} \Theta(p) \approx 0.352$. In this case, it appears that any value of $\Theta(\bar{p})$ below this limit is consistent with equilibrium, $0 \leq \Theta(\bar{p}) \leq \exp(-(\pi + \log 8)/5)$.

6 Buyers' Strikes

One application of the model is to think about whether a movement from one equilibrium to another can be understood as a crisis. We think that an equilibrium with a higher value of $\hat{\beta}$ and hence lower value of $\hat{\theta}$ can be understood as a buyers' strike. As Proposition 3 makes explicit, when $\hat{\beta}$ is higher, fewer investors use their fruit to buy trees. Since the fruit market clears, it follows that the total value of the trees that are sold is lower when $\hat{\beta}$ is higher. In addition, every seller sets a higher price for his tree. While it is not true that every seller is less likely to sell his tree, it is the case that sellers with low continuation values are less likely to sell and sellers with high continuation values are more likely to sell. Therefore, the composition of trees sold shifts towards more expensive trees as well. Both forces imply that fewer trees are sold when $\hat{\beta}$ is higher.

From the perspective of an investor in this environment, nothing extrinsic to the environment has changed. However, some investors stop buying assets because they believe that they will get worse assets at any given price. The reduction in demand shifts the incentive of sellers. In particular, since the marginal buyer values assets more, prices are higher. But to discourage sellers with low continuation values from misrepresenting the quality of their assets, liquidity falls at many prices, and so the shift in beliefs has set off a buyers' strike.

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