Unions in a Frictional Labor Market*

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Abstract

We analyze a labor market with search and matching frictions where wage setting is controlled by a monopoly union. Frictions render existing matches a form of firm-specific capital which is subject to a hold-up problem in a unionized labor market. We study how this hold-up problem manifests itself in a dynamic infinite horizon model, with fully rational agents. We find that wage solidarity, seemingly an important norm governing union operations, leaves the unionized labor market vulnerable to potentially substantial distortions due to hold-up. Introducing a tenure premium in wages may allow the union to avoid the problem entirely, however, potentially allowing efficient hiring. Under an egalitarian wage policy, the degree of commitment to future wages is important for outcomes: with full commitment to future wages, the union achieves efficient hiring in the long run, but hikes up wages in the short run to appropriate rents from firms. Without commitment, and in a Markov-perfect equilibrium, hiring is well below its efficient level both in the short and the long run, with endogenous real stickiness in the dynamics of wages.

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1 Introduction

Labor unions play an important role in many labor markets in many countries. There is also a large literature within labor economics studying how union presence influences labor-market

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outcomes. Yet there is relatively little work studying the impact of this institution on the labor market when this market is described as having frictions and featuring unemployment due to these frictions. Since search and matching models have come to play a central role as a workhorse for macroeconomic labor-market analyses, this gap in the literature leaves open important questions. What is the impact of unions on unemployment and wages? How do unions affect how strongly unemployment varies over the business cycle? What institutional settings are desirable, when considering implementing rules regarding union coverage or centralized bargaining between a union and employer representatives?

The economic distortion perhaps most commonly associated with unionized labor markets is an investment hold-up problem: unions discourage investment in firm-specific capital because they appropriate part of the returns to such investment (Grout 1984, Caballero and Hammour 1998). Union commitment to wages matters for the severity of this problem, as it only arises if the union cannot fully commit to wages before investment takes place. Once we introduce frictions into the labor market, it is important to note that labor itself becomes a form of firm-specific capital, affected by a hold-up problem discouraging hiring. This paper studies how this hold-up problem manifests itself in a dynamic infinite-horizon model of frictional labor markets.[1]

Our model can be interpreted as representing either the aggregate labor market or an industry labor market, but in either case we focus on the case of a "large" union, i.e., one where the union has monopoly power over some group of workers. This case is of particular relevance for many European economies, where there is a nationwide union or cooperation/agreements among unions representing different industries. It is also of relevance in other settings where workers cannot easily move across industries, and competition among different unions within an industry is limited. We assume the union to be fully rational, taking job creation into account when making its wage demands, and its objective to be the welfare of all workers covered by union wages (a special case of this objective is the total wage bill).

Our starting point is a view that union operations are governed by a norm of solidarity

[1]The above papers study static models of firm investment in firm-specific capital in general. A paper within the search and matching framework that also considers hold-up, but focuses on its effects on the cross-sectional distribution of productivity, is Davis (2001). His treatment of the union problem also side-steps the dynamic concerns regarding wage-policy emphasized here.

and egalitarianism among workers, which leads us to the assumption that unions impose identical wages for identical jobs and productivities. This view can be motivated in part by the broad empirical evidence documenting that unions compress the distribution of wages. We find that such fairness comes at a non-trivial cost, however, as it leaves the unionized labor market vulnerable to a potentially severe hold-up problem. Under the egalitarian wage policy the union faces a tension between its competing goals of raising wages to extract rents from firms (the hold-up problem), and encouraging job creation for the benefit of workers hit by an unemployment shock. Due to the influence of the first goal, the end result is inefficiently low hiring. Simply relaxing the egalitarian wage policy by allowing a tenure premium in wages, on the other hand, can provide the union sufficient instruments to satisfy the two goals simultaneously: the union extracts rents by setting high wages for senior workers, while encouraging hiring with low wages for junior workers. In fact, in the absence of binding minimum wage restrictions, introducing such a premium can allow the union to avoid the holdup problem entirely, attaining efficient hiring. The model thus implies a rationale for a tenure premium in union wages.

Under the egalitarian wage policy (and the associated hold-up problem), the degree to which the union can commit to future wages becomes qualitatively and quantitatively important for outcomes.[2] If the union is able to fully commit to future wages, it attains an efficient level of unemployment in the long run. In the short run, however, unemployment is inefficiently high because the union uses its market power to raise current wages above the efficient level, in order to extract rents from firms with pre-existing matches. Specifically, we show that labor-market tightness is inefficiently low in the initial period, but efficient from then on. These elements give rise to a time inconsistency: if a union had decided on a commitment plan yesterday, but had the opportunity to revise it today, it would indeed revise it to benefit again from pre-existing matches.

What would happen if the union did not have commitment to future wages? We answer

[2] The degree of commitment to wages is important in hold-up problems in general, with full commitment potentially avoiding the hold-up problem entirely. In the dynamic model with an egalitarian wage policy the situation is more involved, however, because even in the union problem with full commitment there are some workers who were hired in the past and whose wages will, in part, be set after they have already been hired.

this question by analyzing Markov-perfect equilibria.[3] In these equilibria hiring is below its efficient level both in the short and the long run, and the shorter the commitment horizon, the stronger the effect. With an annual commitment horizon, the effects remain quite sizable: unemployment is well above the efficient level, at 8% instead of 5%, with an output loss of 3% of GDP per period.[4] We also find wages in the unionized setting to exhibit an endogenous short-run real-wage stickiness – a possibility that has motivated macroeconomists to study unions as part of the effort to understand the cyclical variability of hours worked (see, e.g., Blanchard and Fischer 1989).

Throughout this analysis, we focus on the Euler equation of the wage-setting union as our analytical work-horse—both for qualitative analysis of the different forces underlying equilibria as well as for numerical computation. The Euler equation of the union without commitment is readily compared to its efficient counterpart, as well as to the Euler equation of the union with commitment, to develop intuition.

In two extensions, we then proceed to consider what kind of institutional settings may be desirable for counteracting the above distortions. We study economies with: i) less than full unionization of workers, and ii) collective bargaining between a centralized labor union and an employers' association. We demonstrate that if workers have strong individual bargaining power, a law requiring universal coverage of union wages can be welfare-enhancing. Vice versa, if workers have low individual bargaining power, outlawing unions can improve welfare. In the second extension, we consider a Nash bargaining game between a labor union and an employers' association. Here, allowing the employers' association positive bargaining power works toward reversing the distortion associated with the commitment problem. We show an illustrative example where a union bargaining power strictly less than (but close to) one leads to an efficient outcome.

[3] Our focus is on differentiable Markov-perfect equilibria. Thus we do not consider other equilibria where history matters, such as sustainable plans equilibria (Chari and Kehoe 1990). Blanchard and Summers (1988) have argued that unions can give rise to multiple equilibria, which may help explain European labor market outcomes.

[4] Our attaching equal weight to employed and unemployed workers in the union objective renders outcomes less severe than if we let "insiders" (the employed) carry more weight than "outsiders" (the unemployed), as in the insider-outsider approach (Lindbeck and Snower 1986). Similarly, we also allow the union some ability to commit to wages within a period, while outcomes would become even more severe if unions could not commit to wages at all, as in the hold-up literature (Grout 1984).

The extent to which unions in reality compress wages or implement tenure premia is of course an empirical question, and one that we do not pursue here. But what does the available evidence suggest? Formal pay scales appear commonplace in union compensation practices, but the empirical evidence on how returns to tenure differ across union and non-union settings is mixed. A number of earlier studies actually report a stronger association between tenure and earnings in non-union settings (see, e.g., Lewis (1986)), perhaps because unions tend to compress the distribution of wages (Card 1996, DiNardo, Fortin, and Lemieux 1996). Estimating returns to tenure is challenging in itself,[5] but the findings are confounded further by the fact that these estimates tend to be biased by worker and job heterogeneity, generally found to be greater in non-union than union settings. Recognizing this, Abraham and Farber (1988) instead find a stronger association between tenure and earnings in the union setting. Topel (1991), on the other hand, finds no significant effect of union status on returns to tenure.

Our paper is organized as follows. Section 2 discusses the related literature. Section 3 analyzes the benchmark model: first a one-period model to set out notation and introduce the key elements, then an infinite-horizon model with commitment, and finally an infinite-horizon model without commitment. Section 4 provides the quantitative analysis and Section 5 concludes. Appendix A considers extensions: partial unionization and collective bargaining.

2 Related literature

Within the literature on labor unions, this paper is most closely related to two strands: i) a set of papers considering the dynamic decision problem faced by a union when labor is subject to adjustment costs, and ii) a set of papers incorporating a union/unions into the Mortensen-Pissarides search and matching framework, largely focusing on static union decision problems.

The first group of papers develops the idea that dynamic concerns become important for

[5] The magnitude of returns to tenure is a debated topic overall, with estimates ranging from modest (Abraham and Farber 1987, Altonji and Shakotko 1987) to significant (Topel 1991), and the debate continuing (Altonji and Williams 2005, Buchinsky, Fougre, Kramarz, and Tchernis 2010).

thinking about union decision-making when labor markets are not fully frictionless. Most relatedly, Lockwood and Manning (1989) and Modesto and Thomas (2001) study union wage-setting in labor markets where firms face adjustment costs to labor,[6] recognizing that the union's ability to commit to future wages matters for outcomes in this case. The simple quadratic adjustment cost framework adopted affords closed-form results which speak to the level of union wage demands, as well as the speed of adjustment in employment, both argued to be greater in a unionized labor market than a non-unionized one. We, on the other hand, study dynamic union decision-making within the context of the Mortensen-Pissarides search and matching model—the modern workhorse model of frictional labor markets—where such adjustment costs are endogenous. This allows us to study the impact of unions on equilibrium unemployment, vacancy creation, output, and welfare, including getting a sense of the magnitudes of these effects in standard parametrizations of the model. The implications for tenure premia are also unique to our work.

The second group of papers develop extensions of the Mortensen-Pissarides model with a union/unions governing wage determination. Perhaps closest in spirit to our paper in this group is Pissarides (1986), which first introduces a monopoly union into the Pissarides (1985) framework, and studies the impact on equilibrium outcomes in the labor market. As the literature following it, that paper focuses on steady states, side-stepping the dynamic issues we focus on here.[7] In their Handbook chapter, Mortensen and Pissarides (1999) discuss hold-up issues and different union objectives and their impact. Due to our focus on dynamics and the role of commitment, we restrict attention in the main part of the paper to a simple union objective (with all workers, employed and unemployed, weighted equally). The hold-up problem here, moreover, differs from the classical Hosios (1990) analysis in that time inconsistency appears with a large player able to control the economy's state variable: employment.

[6]Other papers which feature unions in settings where labor adjustment occurs slowly due to adjustment costs or otherwise, but focus on other issues, include Booth and Schiantarelli (1987), Card (1986), and Kennan (1988).

[7]The steady-state comparisons in Pissarides (1986) cannot be interpreted from a welfare perspective, however, as doing so would require considering the transition from one steady state to another over time. We show that such transitions involve a time-inconsistency problem which gets in the way of efficiency (unless the union is able to get around the hold-up problem).

Garibaldi and Violante (2005) and Boeri and Burda (2009) proceed to study the effects of employment protection policies in a setting where a monopoly/centralized union compresses wages in the face of worker heterogeneity. Ebell and Haefke (2006) study the effects of product market regulation in a setting with firm-level unions and decreasing returns due to monopolistic competition, while Delacroix (2006) extends their framework to allow varying degrees of centralization in wage bargaining, illustrating the U-shaped relationship between the degree of coordination in union bargaining and economic performance, postulated by Calmfors and Driffill (1988). Finally, Acikgoz and Kaymak (2009) study the evolution of skill premia and unionization rates over time in a setting where firm-level unions compress wages across skill groups, and Taschereau-Dumouchel (2011) proceeds to develop a framework where this wage compression is an endogenous outcome of firm-level voting, when technologies exhibit decreasing returns.[8]

3 The benchmark model

This section begins by describing the simple Mortensen-Pissarides search and matching environment we base our analysis on. We then introduce a monopoly union into that framework, and characterize its behavior.

A frictional labor market Time is discrete and the horizon infinite. The economy is populated by a continuum of measure one identical workers, together with a continuum of identical capitalists who employ these workers. All agents have linear utility, and discount the future at rate $\beta < 1$. Capitalists have access to a linear production technology, producing z units of output per period for each worker employed.

The labor market is frictional, requiring capitalists seeking to hire workers to post vacancies. The measure of matches in the beginning of the period is denoted by $n \in [0, 1]$, leaving $1 - n$ workers searching for jobs. Searching workers and posted vacancies are matched according

[8]Further examples of work on unions in a search framework include Mortensen (1989) on multiple equilibria, Burdett and Wright (1993) on the impact of unions under non-transferable utility, and Alvarez and Shimer (2008) on unions in an island framework. Davis (2001) also emphasizes hold-up problems.

to a constant-returns-to-scale matching function $m(v, 1-n)$, where v is the measure of vacancies. With this, the probability with which a searching worker finds a job within a period can be written $\mu(\theta) = m(\theta, 1)$, and the probability with which a vacancy is filled $q(\theta) = m(1, 1/\theta)$, where $\theta = v/(1-n)$ is the labor market tightness. We assume that $\mu'(\theta)$ is positive and decreasing and $q'(\theta)$ negative and increasing. With this, employment equals n plus the measure of new matches, $\mu(\theta)(1-n)$. Jobs are destroyed each period with probability δ. Thus, the measure of matches evolves over time according to the law of motion

$$n_{t+1} = (1-\delta)\underbrace{\left(n_t + \mu(\theta_t)(1-n_t)\right)}_{\text{employed}_t}. \tag{1}$$

Notice that a worker separated after production at t may be re-employed in $t+1$ and not need to suffer unemployment.

In addition to the market production technology, unemployed workers also have access to a home production technology, producing $b(< z)$ units of output per period.

Firms Capitalists operate production through firms, and these firms need to post vacancies in order to find workers, at a cost κ per vacancy. Competition drives profits from vacancy-creation to zero, with firms taking into account the union wage-setting behavior today and in the future. The zero-profit condition thus determines the current labor-market tightness according to current and future wages $\{w_{t+s}\}_{s=0}^{\infty}$ as follows:

$$\kappa = q(\theta_t) \sum_{s=0}^{\infty} \beta^s (1-\delta)^s \left[z - w_{t+s}\right]. \tag{2}$$

An egalitarian labor union Wages are set unilaterally by a labor union, with universal coverage. The union sets wages to maximize the welfare of all workers,[9] with equal pay for

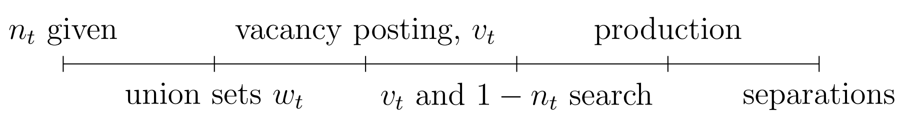
[9] Note that if we normalize $b = 0$, then the union objective becomes the total wage bill.

all those employed. The union objective thus becomes

$$\sum_{t=0}^{\infty} \beta^t \Big[\underbrace{\big(n_t + \mu(\theta_t)(1-n_t)\big)}_{\text{employed}_t} w_t + \underbrace{(1-n_t)(1-\mu(\theta_t))}_{\text{unemployed}_t} b \Big]. \tag{3}$$

The union takes as given the evolution of employment according to equation (1). It also internalizes the effect of its wage-setting decisions on hiring. Therefore, the union's problem is to choose a sequence of wages $\{w_t\}_{t=0}^{\infty}$ to maximize the objective (3) subject to the law of motion (1) and zero profit condition (2). Finally, we also want to make sure that firms, at each point in time, make a non-negative present value of profits from employing a worker, as otherwise they would prefer to end the match. Note, however, that this holds whenever there is positive vacancy posting: firms posting vacancies break even, but that implies that pre-existing matches must have strictly positive value.

Summarizing the events in period t, we have



Given the path of wages $\{w_t\}_{t=0}^{\infty}$, then, equation (2) determines the path of market tightness $\{\theta_t\}_{t=0}^{\infty}$, which in turn determines the evolution of employment.

A non-egalitarian labor union If we relax the equal pay constraint in wage setting by allowing the union to pay different wages to newly hired workers (w_t^n) and workers in existing matches (w_t^e), then the union objective becomes

$$\sum_{t=0}^{\infty} \beta^t \big[n_t w_t^e + \mu(\theta_t)(1-n_t) w_t^n + (1-n_t)(1-\mu(\theta_t)) b \big], \tag{4}$$

and the zero profit condition

$$\kappa = q(\theta_t) \big[z - w_t^n + \sum_{s=1}^{\infty} \beta^s (1-\delta)^s (z - w_{t+s}^e) \big]. \tag{5}$$

In this case we must also impose a separate condition ensuring firms make a non-negative present value of profits on existing workers:

$$\sum_{s=0}^{\infty} \beta^s (1-\delta)^s (z - w^e_{t+s}) \geq 0, \forall t \geq 0. \tag{6}$$

3.1 A one-period example

To illustrate key forces at play, we first consider the impact of the union in a very simple setting: a one-period version of the above economy. Many of the features present here will be present in the subsequent analysis.

A natural starting point is the efficient benchmark—the output maximizing level of vacancy-creation a social planner would choose. Here the planner solves the problem

$$\max_{\theta} \big(\underbrace{n + \mu(\theta)(1-n)}_{\text{employed}}\big) z + \underbrace{(1-n)\big(1-\mu(\theta)\big)}_{\text{unemployed}} b - \underbrace{\theta(1-n)}_{\text{vacancies}} \kappa,$$

taking as given pre-existing matches n. The planner's optimum is characterized by the first-order condition $-\kappa + \mu'(\theta)(z-b) = 0$, which pins down θ independent of n.[10] For concreteness, consider the matching function $m(v,u) = vu/(v+u)$, such that $\mu(\theta) = \theta/(1+\theta)$. In this case the planner's optimum is given by $\theta^p = \sqrt{(z-b)/\kappa} - 1$, with labor-market tightness an increasing function of market productivity. Of course, we must have $z - b > \kappa$ for vacancy creation to be optimal.

The *egalitarian union* instead aims to maximize

$$\big(\underbrace{n + \mu(\theta)(1-n)}_{\text{employed}}\big) w + \underbrace{(1-n)\big(1-\mu(\theta)\big)}_{\text{unemployed}} b,$$

by choice of w and θ, subject to the zero-profit condition: $\kappa = q(\theta)(z-w)$. Here the zero profit condition ensures that firms will be willing to continue to employ the workers in pre-existing matches, as $w \leq z$. To see how this problem relates to the planner's problem, we

[10] In the Mortensen-Pissarides model the path of θ is generally independent of the path of employment—a special case of the block-recursivity property of Menzio and Shi (2010). (The converse is not true, of course.)

can use the zero-profit condition to solve for the wage, as $w = z - \kappa/q(\theta)$, and substitute into the union objective to yield a maximization problem in θ only:

$$\begin{aligned}&\max_{\theta} \big(\underbrace{n + \mu(\theta)(1-n)}_{\text{employed}}\big)\big(z - \frac{\kappa}{q(\theta)}\big) + \underbrace{(1-n)\big(1-\mu(\theta)\big)}_{\text{unemployed}} b \\ &= \max_{\theta} - \underbrace{\frac{n\kappa}{q(\theta)}}_{\text{capitalists' share}} + \underbrace{\big(n + \mu(\theta)(1-n)\big)z + (1-n)\big(1-\mu(\theta)\big)b - \theta(1-n)\kappa}_{\text{planner's objective}},\end{aligned}$$

also taking as given n.

The first line expresses the tradeoff the union faces in choosing θ: increasing θ increases employment, but at the cost of the lost wage income required to raise θ.[11] From the second line, we see that the union objective differs from the planner's objective only by the term $-\frac{n\kappa}{q(\theta)}$. To understand how the two objectives relate to each other, recall that while the planner cares about all agents in the economy, the union only cares about workers. The union objective thus equals the planner's objective less the capitalists' share: (i) profits from new matches, which are zero due to free entry, and (ii) profits from existing matches, which can be expressed as $n(z-w) = \frac{n\kappa}{q(\theta)}$ (using the zero-profit condition).

An interior union optimum is characterized by the first-order condition $-\kappa + \kappa \frac{n}{1-n}\frac{q'(\theta)}{q(\theta)^2} + \mu'(\theta)(z-b) = 0$, which implies that the union's choice of θ *does depend on* n. In our example, an interior union optimum is given by $\theta = \sqrt{1-n}\sqrt{(z-b)/\kappa} - 1$. Labor-market tightness is thus again an increasing function of market productivity, but now decreases in pre-existing matches. Clearly the union implements the socially optimal level of vacancy creation if $n = 0$. But if $n > 0$, the union has an incentive to raise wages above the level consistent with efficient vacancy creation, in order to collect surpluses from firms with existing matches.

[11] This substitution assumes some vacancy creation is optimal. The union could also opt to simply set $w = z$ in the original problem, achieving the value $b + n(z-b)$ for the objective (forgoing vacancy costs entirely). To ensure the solution in the text is optimal, it is necessary to make sure the value of the objective exceeds this value.

The *non-egalitarian union* instead solves the problem

$$\max_{\theta, w^e, w^n} nw^e + \mu(\theta)(1-n)w^n + (1-\mu(\theta))(1-n)b$$
$$\text{s.t. } q(\theta)(z - w^n) = \kappa, \text{ (new matches)}$$
$$w^e \leq z. \text{ (existing matches)}$$

If we allow $w^e \neq w^n$, it is immediately optimal to set $w^e = z$. Substituting this into the union objective then yields the planner objective above, along with the same condition for optimal hiring, $-\kappa + \mu'(\theta)(z - b) = 0$. With this, the wage in new matches is given by $w^n = z - \kappa/q(\theta)$, implying a tenure premium: $w^n < w^e$.

Note that the problem of the non-egalitarian union is essentially a Pareto-problem where the choice variables include "transfers" from capitalists to workers, which may differ across new (w^n) and existing (w^e) matches. The objective is the welfare of workers (equally weighted), and as constraints we require the payoffs of capitalists to be non-negative, for both types of matches. If we additionally impose equal treatment, $w^e = w^n$, then we can drop the second constraint (as it is implied by the first), to arrive at the problem of the egalitarian union above. Thus, the inefficiency in the latter case is entirely due to the constraint to treat identical workers identically.

These distortions arise because search frictions render existing matches a form of firm-specific capital which is subject to a hold-up problem in a unionized labor market. As is typically the case, the degree of commitment to wages is important for the severity of the hold-up problem. In the extreme case, if wages were set after vacancy creation takes place (rather than before), the union would simply set (both) wages equal to z, with no new hiring taking place. The timing here allows the union to commit to wages before vacancy creation, however, and because of this the hold-up problem disappears entirely if we allow wages to differ across new and existing matches. In the egalitarian case, on the other hand, even though the union can commit to the wage before vacancy creation takes place, the presence of the pre-existing matches in the union objective again introduces an incentive for the union to hike up wages toward z. As a result, the union sets the single wage trading off the competing goals of extracting rents on pre-existing matches and encouraging the creation of new ones, unable to avoid distorting hiring in doing so.[12]

The above analysis suggests that tenure premia in union wages may have a natural justification as a means of avoiding the holdup problem associated with a frictional labor market. As shown above, in the absence of a binding lower bound on wages (limiting how low the wages of new workers can be), a tenure premium may even allow the union to attain efficient hiring. While this one-period problem captures the essence of our argument, due to the dynamic nature of the problem we need to think about how this argument plays out in the infinite-horizon model. The next section thus returns to the infinite-horizon setting, where commitment again plays an important role for outcomes.

3.2 The efficient benchmark and a recursive planner's problem

To characterize union wage-setting when the time horizon is infinite, we again begin with the efficient benchmark. The planner now chooses a sequence $\{\theta_t\}_{t=0}^{\infty}$, with $\theta_t \geq 0$, to maximize

$$\sum_{t=0}^{\infty} \beta^t \Big[\underbrace{\big(n_t + \mu(\theta_t)(1-n_t)\big)}_{\text{employed}_t} z + \underbrace{(1-n_t)(1-\mu(\theta_t))}_{\text{unemployed}_t} b - \underbrace{\theta_t(1-n_t)}_{\text{vacancies}_t} \kappa \Big]$$

$$\text{s.t. } n_{t+1} = (1-\delta) \underbrace{\big(n_t + \mu(\theta_t)(1-n_t)\big)}_{\text{employed}_t},$$

with n_0 given.

For what comes later it will be useful to formulate problems recursively. Thus, we begin by writing the planner's problem recursively, and discussing efficient vacancy creation in that context. We then compare to outcomes in the unionized economy.

The recursive form for the planner's problem reads

$$V(n) = \max_{\theta} \big(n + \mu(\theta)(1-n)\big)z + (1-n)\big(1-\mu(\theta)\big)b - \theta(1-n)\kappa + \beta V\big(N(n,\theta)\big), \quad (7)$$

[12] Similarly, if the union placed more weight on workers in pre-existing matches in its objective, it would set an even higher wage than here. In the limit, if it only cared about pre-existing matches, it would set the wage to z, with no new hiring taking place.

where $N(n,\theta) \equiv (1-\delta)\big(n+\mu(\theta)(1-n)\big)$. Notice that the state variable is n, the number of matches at the beginning of the period, and that the control variable—labor-market tightness θ—determines n' according to the law of motion $N(n,\theta)$.

The first-order condition, assuming an interior solution, is

$$\kappa = \mu'(\theta)\big(z - b + \beta(1-\delta)V'\big(n'\big)\big). \tag{8}$$

It equalizes the cost of an additional vacancy, κ, to its benefits: the increase in vacancies increases hiring by $\mu'(\theta)$, with each new worker delivering the flow surplus $z - b$ today together with a continuation value reflecting future flow surpluses. The envelope condition gives the marginal value of a beginning-of-period match, as

$$V'(n) = (1 - \mu(\theta) + \theta\mu'(\theta))\big(z - b + \beta(1-\delta)V'(n')\big). \tag{9}$$

An additional match has the same benefit as above: the flow surplus $z - b$ today and the corresponding continuation value. An increase in beginning-of-period matches increases the planner surplus by this benefit, but there is an additional effect as well: the increase in existing matches hampers hiring by shrinking the pool of unemployed. To see this in the expression, note that the derivative of the matching function with respect to unemployment, $m_u(\theta, 1)$, equals $\mu(\theta) - \theta\mu'(\theta)$.

Eliminating the derivative of the value function in (8), we arrive at the Euler equation

$$\frac{\kappa}{\mu'(\theta)} = z - b + \beta(1-\delta)\big(1 - \mu(\theta') + \theta'\mu'(\theta')\big)\frac{\kappa}{\mu'(\theta')}, \tag{10}$$

where θ is short for the optimal choice of θ given the state variable n. This equation states the efficiency condition for the Mortensen-Pissarides model, solving a tradeoff between the costs and benefits of creating a new match today.

To understand equation (10), note that the cost of creating an additional match today equals the cost of a vacancy, κ, times the measure of vacancies required for one match. Since an increase in vacancies by one unit increases labor market tightness by $1/(1-n)$ units, and an increase in market tightness by one unit gives $(1-n)\mu'(\theta)$ new matches, one new vacancy creates $\mu'(\theta)$ new matches. Hence, the cost of one new match today is $\kappa/\mu'(\theta)$. The benefits include the market production output net of home production output today, $z-b$, as well as what is saved on vacancy creation costs next period. How much is saved? First, note that the net change in matches next period is not simply $1-\delta$. Although share $1-\delta$ of the newly created matches survive to the next period, the increase in matches also shrinks the pool of unemployed, so that any planned vacancy-creation next period will yield fewer matches. For each worker now out of the unemployment pool, there is a decrease in new matches given by $m_u(\theta', 1) = \mu(\theta) - \theta\mu'(\theta)$. Creating an additional match today thus leads to a net increase in matches next period of $(1-\delta)(1-\mu(\theta') + \theta'\mu'(\theta'))$, with each additional match worth $\kappa/\mu'(\theta')$ consumption units.

From the Euler equation, we notice a familiar feature of the benchmark search and matching model: it does not feature the state variable n explicitly. Only market tightness today and tomorrow appear, so that a natural solution is a constant tightness independent of n. It is straightforward to show that the Bellman equation is solved by a value function V that is linear in n, and that the efficient allocation thus features a constant tightness $\theta_t = \theta^p \; \forall t \geq 0$, independent of n.

3.3 An egalitarian union with commitment

Turning to the unionized labor market, we begin with the case of the egalitarian union, choosing a single sequence of wages $\{w_t\}_{t=0}^{\infty}$ to maximize the objective (3) subject to the law of motion (1) and zero profit conditions (2).

In order to relate the union problem to the planner's problem, we again use the zero profit conditions to rewrite the union objective. To this end, note that the union's choice of a sequence of wages determines, at each instant, the present value of wages workers expect to earn over an employment spell, as $W_t = \sum_{s=0}^{\infty} \beta^s (1-\delta)^s w_{t+s}$. The sequence of these present values $\{W_t\}_{t=0}^{\infty}$ then pins down the sequence $\{\theta_t\}_{t=0}^{\infty}$ through the zero-profit conditions, assuming some vacancy creation occurs each period. Conversely, given a sequence $\{\theta_t\}_{t=0}^{\infty}$, one can back out per-period wages by first using the zero-profit condition to find the present value of wages W_t each period, and then computing wages as $w_t = W_t - \beta(1-\delta)W_{t+1}$.

Using the zero-profit condition to eliminate wages, the union objective (3) can be written as (see Appendix B):

$$-\frac{n_0\kappa}{q(\theta_0)} + \sum_{t=0}^{\infty}\beta^t\left[\left(n_t + \mu(\theta_t)(1-n_t)\right)z + (1-n_t)(1-\mu(\theta_t))b - \theta_t(1-n_t)\kappa\right], \qquad (11)$$

revealing an identical objective to that of the planner except for the first term. This term—familiar from the one-period example—reflects the share of the present discounted value of output accruing to capitalists. To see this, note that the capitalists' share, i.e., the present value of profits to firms, can be written as

$$n_0\sum_{t=0}^{\infty}\beta^t(1-\delta)^t[z-w_t] + \sum_{t=0}^{\infty}\beta^t[\mu(\theta_t)(1-n_t)\sum_{s=0}^{\infty}\beta^s(1-\delta)^s[z-w_{t+s}] - \theta_t(1-n_t)\kappa]. \quad (12)$$

Here the first term captures the present value of profits on existing matches, and the second those on new vacancies created in periods $t = 0, 1, \ldots$. The expression reduces to representing initial matches only, however, as free entry drives the present value of profits to new vacancies to zero.[13] Pre-existing matches, on the other hand, are due a strictly positive present value of profits, because these firms paid the vacancy cost in the past, anticipating positive profits in the future to make up for it. Using the zero-profit condition, this remaining present value can be expressed as $n_0\kappa/q(\theta_0)$.

The union objective (11) reflects the fact that while the planner maximizes the present discounted value of output, the union only cares about the workers' share of it. As a result, the union will have an incentive to appropriate some of this present value from capitalists by raising wages above the efficient level—and this is exactly how the solutions to the two problems will differ.

Proposition 1. *If the union is able to commit to future wages, hiring is efficient after the initial period. In the initial period, hiring is efficient if* $n_0 = 0$ *and below efficient if* $n_0 > 0$.

Note that after the initial period, the union effectively solves the planner's problem (7), and

[13]We can write the second term in equation (12) as $\sum_{t=0}^{\infty}\beta^t(1-n_t)\theta_t[q(\theta_t)\sum_{s=0}^{\infty}\beta^s(1-\delta)^s[z-w_{t+s}]-\kappa]$, which equals zero due to the free entry condition (2).

consequently chooses the planner's solution $\theta_t = \theta^p \ \forall t \geq 1$. In the initial period, however, the union chooses θ_0 to maximize

$$-\frac{n_0\kappa}{q(\theta_0)} + \big(n_0 + \mu(\theta_0)(1-n_0)\big)z + (1-n_0)\big(1-\mu(\theta_0)\big)b - \theta_0(1-n_0)\kappa + \beta V\big(N(n_0,\theta_0)\big), \tag{13}$$

where n_0 is given, and V solves the planner's problem (7).[14]

Deriving the optimality condition for this initial period is straightforward using the same methods as above. It becomes

$$[1 - \frac{n_0}{1-n_0}\frac{q'(\theta_0)}{q(\theta_0)^2}]\frac{\kappa}{\mu'(\theta_0)} = z - b + \beta(1-\delta)\big(1-\mu(\theta^p) + \theta^p\mu'(\theta^p)\big)\frac{\kappa}{\mu'(\theta^p)}, \tag{14}$$

where we have used the fact that in subsequent periods we will have the efficient market tightness θ^p. Comparing to the efficiency condition (10), the cost of creating an additional match today (on the left) is higher for the union than for the planner. This occurs because in order to increase hiring, the union must lower wages, giving up some of the surplus it could have appropriated from firms with existing matches. Moreover, the more existing matches there are, the greater this additional cost.

Using the efficiency condition (10), we can further rewrite equation (14) as

$$[1 - \frac{n_0}{1-n_0}\frac{q'(\theta_0)}{q(\theta_0)^2}]\frac{1}{\mu'(\theta_0)} = \frac{1}{\mu'(\theta^p)}.$$

Because $q'(\theta) < 0$ and $\mu'(\theta)$ is decreasing, this equation implies: (i) a lower value of θ_0 in the initial period than later on and (ii) the more initial matches, the stronger this effect. Thus, initial market tightness depends negatively on the measure of pre-existing matches. This is a key feature of the model, which becomes even more important when unions do not have

[14] Like in the one-period problem, using the zero profit condition to substitute out wages assumes positive vacancy creation each period. Here the union could again choose to set initial wages to shut down hiring in the first period completely, and then resume efficient hiring from lower initial employment in the following period. It is straightforward to calculate numerically at what employment level such extreme confiscating becomes optimal. Calculations show that this occurs at lower employment levels if the wage-setting period is short, because shutting down hiring for a short time is not that costly relative to the rewards. When the wage-setting period is on the order of a year, these costs become much more severe, however, discouraging from the use of this policy.

commitment.

That the outcome in the initial period differs from later periods reflects a time inconsistency issue in the union wage-setting problem. If the union were to re-optimize after the initial period, it would face a different objective and choose a different path of wages. While the union can thus get relatively close to the efficient outcome when it can commit, this immediate time inconsistency begs the question: what happens if the union cannot commit to future actions? To study time-consistent union decision making we next turn to a game-theoretic setting, which will be based on the recursive formulation of the union problem we set up above.

3.4 An egalitarian union without commitment

The union problem (13) suggests that if the union were to re-optimize at any date, its choice of initial θ would depend on n, the measure of matches in the beginning of the period. In particular, a higher n should imply a lower θ. How would outcomes change if the union could not commit to not re-optimizing? We study this question by focusing on (differentiable) Markov-perfect equilibria with n as a state variable. That n is a payoff- and action-relevant state variable should be clear from the problem under commitment.[15] In a Markov-perfect equilibrium, the union anticipates its future choices of θ to depend (negatively) on n, a relationship we label $\Theta(n)$. Our task is now to characterize $\Theta(n)$.

The function $\Theta(n)$ solves a problem similar to (13), namely

$$\Theta(n) \equiv \arg\max_{\theta} -\frac{n\kappa}{q(\theta)} + \big(n + \mu(\theta)(1-n)\big)z + (1-n)(1-\mu(\theta))b - \theta(1-n)\kappa + \beta\tilde{V}\big(N(n,\theta)\big), \tag{15}$$

where the continuation value $\tilde{V}$ satisfies the recursive equation

$$\tilde{V}(n) = \big(n + \mu(\Theta(n))(1-n)\big)z + (1-n)(1-\mu(\Theta(n)))b - \Theta(n)(1-n)\kappa + \beta\tilde{V}\big(N(n,\Theta(n))\big). \tag{16}$$

[15] One can add states, representing histories of past behavior, but we do not consider such equilibria here.

Here, the union recognizes that its future actions will follow $\Theta(n)$, and this is reflected in the continuation value $\tilde{V}(n)$. Because $\Theta(n)$ will generally not be efficient, $\tilde{V}$ will not equal V, the continuation value under commitment.

A *Markov-perfect equilibrium* is defined as a pair of functions $\Theta(n)$ and $\tilde{V}(n)$ solving (15)–(16) for all n. We will assume that these functions are differentiable and characterize equilibria based on this assumption. We discuss issues of existence and uniqueness/multiplicity of equilibria in Section 4 below.

From equation (15), the first-order condition for market tightness becomes

$$[1 - \frac{n}{1-n}\frac{q'(\theta)}{q(\theta)^2}]\kappa = \mu'(\theta)\big(z - b + \beta(1-\delta)\tilde{V}'\big(n'\big)\big), \tag{17}$$

and the equation paralleling the envelope condition—now not formally an envelope condition since the union does not agree with its future decisions—becomes

$$\begin{aligned}\tilde{V}'(n) =&(1-\mu(\theta)+\theta\mu'(\theta))\big(z-b+\beta(1-\delta)\tilde{V}'(n')\big)\\ &+\mu'(\theta)\big(\Theta'(n)(1-n)-\theta\big)\big(-\frac{n}{1-n}\frac{q'(\theta)}{q(\theta)^2}\frac{\kappa}{\mu'(\theta)}\big).\end{aligned} \tag{18}$$

Equation (18) is derived by differentiating equation (16), and using equation (17) to arrive at a formulation close to the equivalent condition (9) for the planner. Compared to the planner's envelope condition, this equation includes some additional terms, which appear because the envelope theorem does not apply, reducing the value of additional initial matches n. The current union regards next period's union as setting θ too low, and because θ is lower the greater is n, additional initial matches are less valuable for the union.

We can further combine the above two equations to eliminate $\tilde{V}'$, obtaining

$$\underbrace{[1-\frac{n}{1-n}\frac{q'(\theta)}{q(\theta)^2}]\frac{\kappa}{\mu'(\theta)}}_{\text{cost of match today}} = z-b+\beta(1-\delta)[\big(1-\mu(\theta')+\theta'\mu'(\theta')\big)\underbrace{[1-\frac{n'}{1-n'}\frac{q'(\theta')}{q(\theta')^2}]\frac{\kappa}{\mu'(\theta')}}_{\text{value of match tomorrow}}$$
$$+\underbrace{\mu'(\theta')(\Theta'(n')(1-n')-\theta')(-\frac{n'}{1-n'}\frac{q'(\theta')}{q(\theta')^2})\frac{\kappa}{\mu'(\theta')}}_{\text{loss in value from lack of commitment}}], \tag{19}$$

which is a *generalized Euler equation.* It is a functional equation in the unknown policy function Θ, where the derivative of Θ appears. The equation is written in a short-hand way: θ is short for $\Theta(n)$, θ' is short for $\Theta(N(n,\Theta(n)))$, and n' is short for $N(n,\Theta(n))$. The task, then, is to find a function Θ that solves this equation for all n. Note that in contrast to the planner's Euler equation, n appears non-trivially in this equation and will generally matter for the tightness—it is easily verified that a constant Θ will not solve the equation.

In terms of interpretation, this equation, like the planner's Euler equation (10), represents the tradeoff between the costs and benefits of creating matches today. The cost of an additional match for the union differs from the cost for the planner, however: in addition to the increase in vacancy costs $\kappa/\mu'(\theta)$, the union also takes into account that increasing hiring requires reducing wages, thereby giving up some of the surplus it could have appropriated from capitalists, as captured by the term $-\frac{n}{1-n}\frac{q'(\theta)}{q(\theta)^2}\frac{\kappa}{\mu'(\theta)}$. The present value of an additional worker must therefore, from equation (17), be higher in the unionized economy than what is efficient. This additional cost appears also in the Euler equation (14) for the union with commitment, but here it appears both today and tomorrow symmetrically, unlike in the commitment solution where tomorrow's union mechanically carries out the orders of today's plan.

Beyond this difference, here the union also takes into account its inability to commit to future wages: more matches tomorrow will reduce hiring, as the union will raise wages in response. To see this, note that the measure of vacancies can be written as $\Theta(n)(1-n)$ and its derivative with respect to initial matches n as $\Theta'(n)(1-n)-\Theta(n)$. A marginal increase in matches thus reduces new hiring by $\mu'(\theta)(\Theta'(n)(1-n)-\Theta(n))$, with each lost worker valued at the size of the distortion in the union objective—the marginal surplus appropriated from capitalists.

For the present model it is hard to establish, theoretically, that $\Theta(n)$ is indeed decreasing. In the one-period example of Section 3.1 we saw that Θ becomes a decreasing function of n, and in our numerically solved examples below, this feature is always present.[16] What is possible to show for the infinite-horizon case, however, is that whenever $\Theta(n)$ is decreasing, steady-state market tightness is strictly below its efficient level:

Proposition 2. *If $\Theta(n)$ is decreasing in n, then the steady-state market tightness, θ, in the unionized economy (without commitment) is strictly below its efficient level.*

It follows that steady-state unemployment in the unionized economy is strictly above its efficient level.

3.5 A non-egalitarian union

Finally, we turn to the case of the non-egalitarian union, which chooses two sequences of wages, $\{w_t^n\}_{t=0}^{\infty}$ and $\{w_t^e\}_{t=0}^{\infty}$, to maximize the objective (4) subject to the law of motion (1), zero profit conditions (5), and constraints (6) ensuring firms make non-negative profits on existing workers at all times.

In setting the wages of existing workers, the best the union can do is to set $w_t^e = z$ each period, leaving firms with zero surplus on existing matches. From the zero profit condition then, we have that $w_t^n = z - \kappa/q(\theta_t)$, $\forall t \geq 0$. Using this expression to substitute out wages in the union objective, it is easy to see that the union problem becomes identical to the planner problem, thus leading to efficient hiring: $\theta_t = \theta^p$, $\forall t \geq 0$. The solution therefore involves a constant (efficient) market tightness θ^p over time, as well as constant wages for new and existing workers: $w_t^n = z - \kappa/q(\theta^p)$ and $w_t^e = z$ $\forall t \geq 0$, featuring a tenure premium.

We therefore conclude that in the infinite horizon setting as well, the union may be able to attain efficient hiring with the use of a wage tenure premium. A potential concern with this wage policy is that the implied wages of new workers may be quite low in the infinite

[16] We also have not been able to find an example where Θ is not decreasing.

horizon model, however, as firms must make the entire present value of profits associated with efficient hiring in the first period of the match. The implied wage of new workers might be negative, for example, potentially perceived infeasible. If there is a lower bound on wages $\underline{w}(< z)$, and that lower bound is binding, such that $z - \kappa/q(\theta^p) < \underline{w}$, then hiring will be distorted down. The union wage policy will still involve a tenure premium, but in this case the market tightness solves $q(\underline{\theta}_t)(z - \underline{w}) = \kappa \ \forall t \geq 0$.

In sum, wage solidarity comes at a cost in this economy, due to the hold-up problem associated with frictional labor markets. This conclusion suggests a role for tenure premia in union wages as a means to avoid the resulting distortions in hiring. And yet, the empirical evidence on how returns to tenure depend on union status does not point to clearly greater returns in unionized settings than otherwise. Is this simply due to the measurement problems involved in the empirical work? Or are the distortions perhaps too insignificant in magnitude to warrant giving up (the benefits underlying) wage solidarity? To shed light on this question, the next section turns to the task of solving for equilibrium outcomes under wage solidarity when the union cannot commit to future wages.

4 Quantitative results: comparative statics and comparative dynamics

The presence of an egalitarian union affects the levels of unemployment, wages, and output in the economy, distorting outcomes from efficiency. In this section we parameterize the model to shed light on how large an impact this has on the economy. We also consider an extension with stochastic shocks to productivity and ask whether, in this model, shock amplification is significantly different than in the standard model.

4.1 Wages, unemployment, and output in steady state

How does the presence of the egalitarian union in the labor market affect the *levels* of wages, unemployment, and output? The theory tells us that the answer hinges on the union's ability to commit to future wages. If the union can commit, the unionized economy attains efficiency in the long run. If the union cannot commit, the theory leads us to expect higher wages and unemployment, and consequently lower output, in the unionized economy than what would be efficient, both in the short and the long run.

Calibration We parameterize the model such that the efficient outcome represents the US labor market, as calibrated by Shimer (2005), and study how introducing the union into this economy changes outcomes.[17] In doing so, we adopt an annual frequency, to reflect the annual wage-setting practices observed (we also consider other frequencies below). We first set the time discount rate to correspond to a 5 percent annual rate of return, with $\beta = 1/1.05$. We normalize labor productivity to $z = 1$ and set $b = 0.4$ (we consider higher values of b as well). We depart from Shimer's specification slightly by adopting the matching function $m(v, u) = \mu_0 vu/(v + u)$, used by, e.g., den Haan, Ramey, and Watson (2000). This form is better suited for the discrete-time setting than a Cobb-Douglas functional form because it helps ensure that matching probabilities remain between zero and one. We pin down the remaining parameters δ, μ_0 and κ as follows. First, attaining an average duration of employment of 2.5 years requires a separation rate of $\delta = 0.40$. Second, to also be consistent with a steady-state unemployment rate of 5 percent, the average job-finding rate must be $\mu(\theta) = 0.88$. Finally, to also match the slope of the Beveridge curve, documented by Shimer (2007) to equal -1, this requires setting $\mu_0 = 1.01$ and a steady-state value of $\theta = 7.17$.[18] The latter can be achieved by setting $\kappa = 0.010$.

Numerical solution technique The planner's problem, as well as the case of a union with commitment, can be solved almost in closed form. Solving for the union's behavior when it cannot commit is more challenging, however, with several issues to bear in mind. On the one hand, there are few results available on equilibrium existence for differentiable Markov-perfect equilibria. Moreover, differentiable equilibria may not be unique. And further, non-differentiable equilibria may exist as well.[19] Clearly, one needs to proceed with caution

[17] Shimer (2005) calibrates a model with a decentralized labor market, but the calibration strategy causes this equilibrium outcome to coincide with the socially optimal one.

[18] This holds both based on the vacancy data from JOLTS as well as the longer time series of help-wanted advertising from the Conference Board, with $d \log v/d \log u \approx -1$.

[19] For examples where no differentiable equilibria exist but there exists a non-differentiable equilibrium see, e.g., Krusell, Martin, and Rios-Rull (2010); for cases with a continuum of non-differentiable equilibria along

and be prepared to use several different solution techniques. The results we present in the tables and figures below use the methods in Krusell, Kuruscu, and Smith (2002) and rely on approximating the equilibrium function Θ with polynomials of increasingly higher order. However, we have also tried a number of alternative methods, with very similar quantitative results. We discuss these issues in more detail in Appendix C.

Results Table 1 reports the steady-state levels of the wage, unemployment, vacancies, market tightness, and output. The table compares steady-state outcomes in a unionized economy where the egalitarian union cannot commit, to the efficient steady state.[20] Without commitment, the union's incentive to raise wages leads to a 1.3 percent increase in steady-state wages,[21] which leads to a 34 percent reduction in firm profits per worker. This reduction in profits then leads to a 38 percent drop in market tightness, along with a 60 percent increase in unemployment, and 36 percent drop in vacancies. Finally, the reduction in employment results in a 3 percent drop in per-period output.

Table 1: Effect of union on levels

Level	Efficient	Union	Union impact
Wages	0.96	0.98	+1.3%
Unemployment	0.05	0.08	+60%
Vacancies	3.08	1.98	-36%
Tightness	7.17	4.43	-38%
Output	0.95	0.92	-3.2%

Notes: The table reports steady-state values. In the case of efficiency, the wage refers to one which would implement the efficient allocation. NB: Market tightness refers to the vacancy-searcher ratio, $v/(1-n)$.

As a robustness check, we provide results in Table 2 for a higher value of home production, with very similar results.

The role of the commitment horizon The recursive formulation assumes the union can commit to current period wages, but not beyond. This suggests that the lack of commitment becomes less of an issue as the period length increases. By adjusting the discount rate β,

with one or more differentiable equilibrium, see Krusell and Smith (2003) or Phelps and Pollak (1968).

[20]This efficient steady state is also identical to: (i) the long run outcome in a unionized economy where the egalitarian union can commit, and (ii) the steady state in Shimer's (2005) decentralized model calibrated to the US labor market.

[21]Relative to the wage which would implement the efficient level of vacancy creation, i.e., the efficient market tightness θ.

Table 2: Effect of union on levels, higher b

Level	Efficient	Union	Union impact
Wages	0.98	0.98	+0.8%
Unemployment	0.05	0.08	+60%
Vacancies	3.08	1.98	-36%
Tightness	7.17	4.43	-38%
Output	0.95	0.92	-3.2%

Notes: The table reports steady-state values. Here $b = 0.6$ and, to maintain efficient unemployment at the 5 percent level, $\kappa = 0.007$. NB: Market tightness refers to the vacancy-searcher ratio, $v/(1-n)$.

the separation rate δ, and the matching function coefficient μ_0, one can examine different period lengths, and hence different degrees of commitment. Table 3 reports the results. On the one hand, we can see that moving from the annual horizon to a shorter, semi-annual horizon, exacerbates the negative effects of limited commitment significantly: employment and output fall by as much as 4 percentage points, with unemployment increasing by about the same, from 8 to 12 percent. On the other hand, moving to the infinite horizon limit would increase employment and output by 3 percentage points, with unemployment falling from 8 to 5 percent.[22]

Table 3: Role of commitment horizon

Level	6 months	12 months	Efficient
Unemployment	0.12	0.08	0.05
Vacancies	0.42	1.98	3.08
Tightness	1.42	4.43	7.17
Output	0.88	0.92	0.95

Notes: The table reports steady-state values. The annual horizon corresponds to our baseline calibration, for the semiannual $\delta = 0.2$ and, to maintain efficient unemployment at the 5 percent level, $\kappa = 0.042$. NB: Market tightness refers to the vacancy-searcher ratio, $v/(1-n)$.

[22]It is intuitive that if the period length approaches zero, the lack of union commitment will lead to 100% unemployment. It is not exactly true that if the period length approaches infinity, the commitment solution obtains, however, as the commitment solution generally involves a time-varying policy, while here policy is fixed within a period.

4.2 Welfare comparisons

The egalitarian union maximizes the welfare of all workers in the economy, thus internalizing the general equilibrium effects of its wage demands. Nevertheless, the unionized economy generally departs from efficiency because the union cannot differentiate between new and existing workers when setting wages. While this occurs even in the simple one period example, in the dynamic model another source of inefficiency appears whenever the union lacks commitment: there is a long-run loss from suboptimal job creation.

How large are the welfare losses resulting from the union presence? To shed light on this question, we study the transitional dynamics of an economy with an egalitarian union which cannot commit to future wages. Starting from steady state, we ask: (i) what would happen if the union gained commitment, and (ii) how do these outcomes differ from the efficient response? Figure 1 illustrates the responses of employment, market tightness and wages in these two cases. As is clear from the pictures, the dynamics of θ reflect our analytical results above: with sudden commitment the union would maintain a low θ in the initial period—it is slightly above the no-commitment steady-state starting point—but then have a fully efficient θ in following periods.[23] Consequently, the dynamics are rather fast, in the sense that in a few periods the efficient and commitment-union economies both have employment very close to the efficient steady-state rate. For purposes of illustration, the figure contrasts outcomes in the annual calibration (on the right) with those in a monthly calibration (on the left). The difference between the efficient response and the commitment-union response is naturally greater in the annual calibration, where the initial period is longer.

How large are the effects on welfare? The present value of output at time zero on these three transition paths are as follows: in the annual calibration, the planner's response yields a present value of 19.95, attaining commitment gives a present value of 19.91, while remaining at no commitment 19.19. In terms of the per-period increase in output from attaining efficiency, these figures translate to 3.25%, while the increase from attaining commitment

[23]From the Euler equations for the commitment and no-commitment cases, a comparison reveals that it is not clear that the dynamics will be monotone. It turns out to be the case in the graph but it could have turned out instead that initial tightness would decrease slightly the first month before jumping up to the steady-state level.

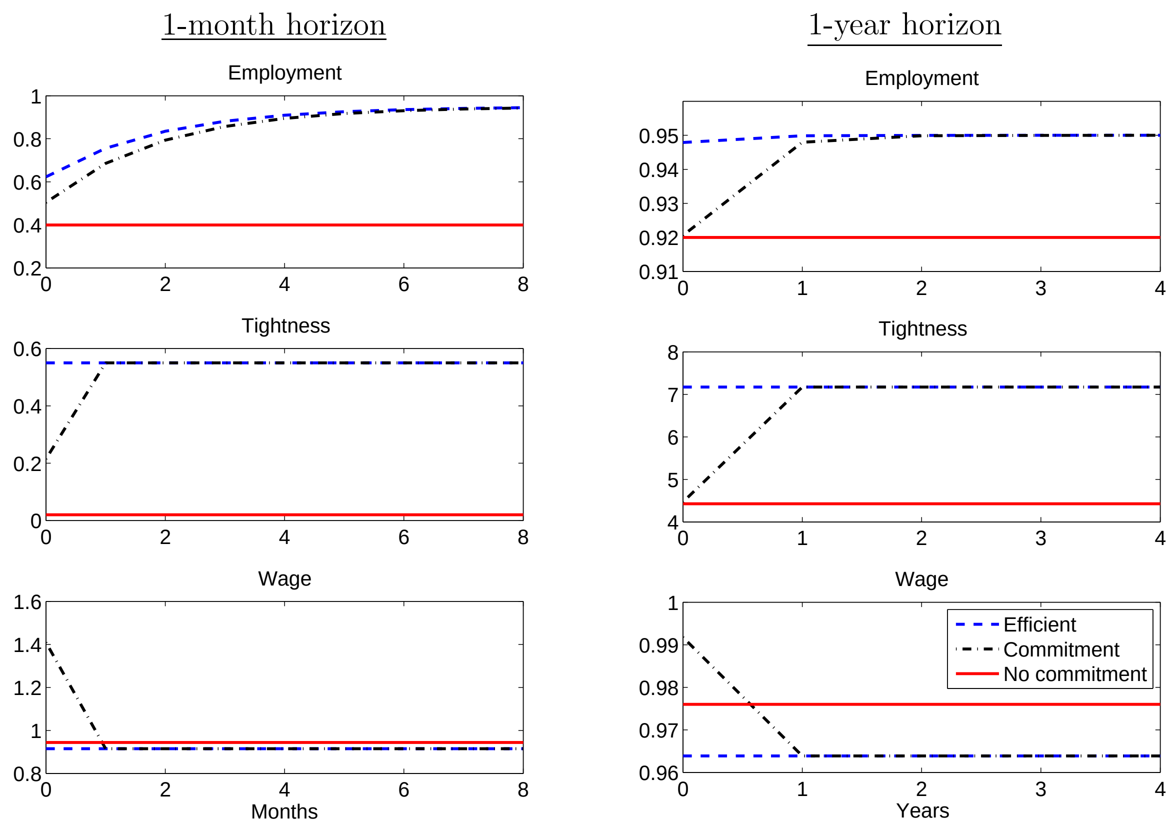


Figure 1: Adjustment dynamics when union gains commitment versus efficient response
Notes: The figure plots adjustment dynamics starting from the steady state where the union cannot commit. The figure shows the planner's response, the union response if it gained commitment, and the union response if it did not.

is 3.10%. For comparison, in the monthly calibration these numbers are 38.6% and 38.5%, respectively. Attaining commitment leads to non-trivial welfare gains in both cases, but the gains are much larger in the monthly calibration because the no-commitment outcome is substantially worse in that case. The difference between attaining commitment and attaining efficiency is larger in the annual calibration, however, as the initial adjustment period is longer in that case.

In two extensions in the appendix to this paper we also look at welfare in slightly different institutional settings. Focusing on the case of an egalitarian wage policy, we consider economies with: i) less than full unionization of workers, and ii) collective bargaining between a centralized labor union and an employers' association. In the first extension, we demonstrate that if workers have strong individual bargaining power, a law requiring universal coverage of union wages can actually be welfare-enhancing. However, if workers have low individual bargaining power, outlawing unions can improve welfare. In the second extension, we consider a Nash bargaining game between a labor union and an employers' association, showing that allowing the employers' association positive bargaining power works toward reversing the distortion associated with the commitment problem. We show an illustrative example where a union bargaining power strictly less than (but close to) one leads to an efficient outcome.

4.3 Aggregate shocks

An important reason macroeconomists have been interested in labor unions is the notion that unions create rigidity in wages, which may help reconcile the large variation in employment over the business cycle with macroeconomic theory (see, e.g., Blanchard and Fischer 1989). What does our theory of unions imply about the responses of wages, vacancy creation, and unemployment to shocks? The dynamics of the model under efficiency are well known, but how do these dynamics change when the labor market has an egalitarian union that cannot commit to future wages? To answer this question, one can study deterministic transitions to steady state. However, it appears more interesting to compare economies that actually feature recurring fluctuations.

One could think of various kinds of shocks perturbing the economy over time. For purposes of illustration, the most obvious shock to consider is one to productivity z. It is straightforward to extend the setup above to allow z to follow a Markov process. A union that cannot commit to future wage setting in this environment will, as in the analysis above, play a dynamic game with its future counterparts, though the game here will be stochastic. As before, it is natural to focus on Markov-perfect equilibria. Thus, $\Theta(n, z)$ now depends on productivity, as

$$\begin{aligned}\Theta(n, z) \equiv \arg\max_{\theta} -\frac{n\kappa}{q(\theta)} + \big(n + \mu(\theta)(1-n)\big)z + (1-n)(1-\mu(\theta))b - \theta(1-n)\kappa \\ +\beta E_z \tilde{V}\big(N(n, \theta), z'\big),\end{aligned}$$

where the continuation value $\tilde{V}$ satisfies the recursive equation

$$\tilde{V}(n,z) = \big(n + \mu(\Theta(n,z))(1-n)\big)z + (1-n)(1-\mu(\Theta(n,z)))b - \Theta(n,z)(1-n)\kappa \\ + \beta E_z \tilde{V}\big(N(n,\Theta(n,z)),z'\big).$$

It is straightforward, along the lines above, to derive the generalized Euler equation for this case as well. It reads

$$[1 - \frac{n}{1-n}\frac{q'(\theta)}{q(\theta)^2}]\frac{\kappa}{\mu'(\theta)} = z - b + \beta(1-\delta)E_z[\big(1-\mu(\theta') + \theta'\mu'(\theta')\big)\big[1 - \frac{n'}{1-n'}\frac{q'(\theta')}{q(\theta')^2}]\frac{\kappa}{\mu'(\theta')} \\ + \mu'(\theta')\big(\Theta_n(n',z')(1-n') - \theta'\big)[-\frac{n'}{1-n'}\frac{q'(\theta')}{q(\theta')^2}]\frac{\kappa}{\mu'(\theta')}],$$

thus differing only in that there is an expectations operator in front of future payoffs.

The model is calibrated as the deterministic one, and our numerical method easily extended to cover the shock case.[24]

We first look at impulse responses. Figure 2 plots the impulse responses of wages, market tightness, unemployment, and output, comparing the unionized economy (solid line), to the efficient outcome (dashed line). Note that the wage response in the efficient outcome refers to the wage which would implement the efficient allocation, i.e., which gives firms exactly the amount of surplus in matching to induce them to create the efficient measure of vacancies. The right panel displays the annual calibration, while the left panel again highlights the effects by displaying a monthly calibration instead.

As the figure shows, in the short run, the union acts so as to introduce "real wage stickiness" into the dynamics, relative to the efficient benchmark. When productivity increases, we observe an on-impact increase in wages as usual. But the increase in productivity also leads to an increase in employment over time, and while this increase in employment plays out, the union's distortionary motive to hike up wages strengthens. As a result, wages continue to rise over time, generating the lagged response in the figure. The lagged wage response amplifies the short-run response of market tightness to the shock, leading to a greater increase in

[24]See Appendix C for details. In brief, the numerical solution is recursive: one can first solve for deterministic dynamics in the state n and, as a function of that, for responses to z.

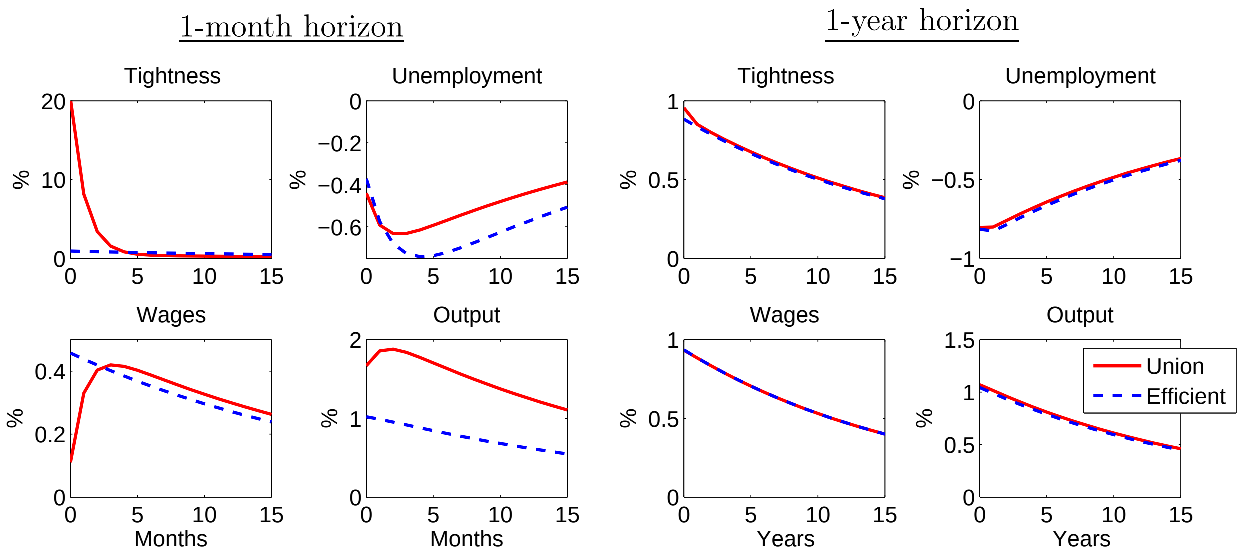


Figure 2: Impulse responses: efficient versus union

Notes: The figure plots impulse responses to a one percent positive productivity shock, when labor productivity follows an AR(1) process (see Table 4 for more).

employment and output as a result. In examining the responses it is useful to keep in mind that the figure plots percentage responses, while the steady states across the two economies are quite different. Because of this the percentage response of unemployment in the figure, for example, is not amplified, because the unionized economy has a substantially higher unemployment rate.[25]

Table 4 below reports simulated moments, quantifying the changes in volatility due to union influence. As the impulse responses indicated, the unionized economy displays greater volatility in a number of variables. As expected, the effects are less striking in magnitude for the annual horizon, however.

5 Conclusions

Labor market frictions turn existing employment relationships into a form of firm-specific capital, which is subject to a hold-up problem in a unionized labor market. This paper has studied how this hold-up problem manifests itself in a dynamic infinite-horizon search

[25] As the figure shows, beyond the short-run wage stickiness, wages are actually slightly more variable in the unionized economy. This is in line with the intuition that the union distortion, which raises wages, should be stronger when employment is higher.

Table 4: Effect of union on volatility

	1-month horizon			1-year horizon		
Volatility	Efficient	Union	Union impact	Efficient	Union	Union impact
Unemployment	0.87	0.72	-17%	0.85	0.84	-1.4%
Vacancies	0.61	10.9	+1700%	0.86	0.91	+5.9%
Tightness	0.96	10.9	+1000%	0.88	0.94	+6.6%
Output	1.04	2.05	+97%	1.04	1.07	+2.7%

Notes: The table reports standard deviations of model variables relative to the standard deviation of labor productivity, based on simulated data from the model, logged and filtered. The monthly series are (after aggregating to quarterly) filtered with HP(1600), while the annual series are filtered with HP(100). The productivity process is an AR(1) such that the aggregated, logged and filtered series has standard deviation 1.62% and persistence 0.47 at the annual frequency, consistent with BLS data, and respectively 1.30% and 0.76 at the quarterly frequency.

and matching model of frictional labor markets. We find that, even though solidarity and egalitarianism may appear to be important values influencing union operations, an egalitarian wage policy can be costly, as it leaves the unionized labor market vulnerable to the distortions stemming from the hold-up problem. We show that relaxing such a policy by allowing a tenure premium in wages can lead to significant welfare gains. We also show that how severe the effects of the hold-up problem are depends importantly on the degree of commitment the union has to future wage policy. With full commitment to future wages, the union quickly attains efficient hiring, despite distortions in the short run. Characterizing equilibrium outcomes when the union does not have commitment to future wage policy is technically clearly more challenging. Focusing on Markov-perfect equilibria, we illustrate the tradeoffs with the help of Euler equations, as well as solving for outcomes using state-of-the-art numerical approaches. We find quantitatively significant distortions in steady-state unemployment and output, as well as endogenous short-run stickiness in union wages.

Our paper offers an argument for tenure premia in union wages. This may come across as an obvious part of union compensation practices, but note that our argument favoring tenure premia applies to workers with identical productivity. In practice the increase in union pay over tenure surely reflects at least in part productivity improvement on the job. Further, and perhaps surprisingly, the empirical evidence on how tenure premia differ across union and non-union settings remains inconclusive. Given the potential costs of wage solidarity discussed, this raises the question of what the motives supporting solidarity are. Here our work comes to contact with a well-known tradition in macroeconomics which seeks to understand the observed behavior of wages emphasizing notions of fairness and morale as playing an important role in wage-setting practices. Developing these ideas further appears both challenging and potentially quite rewarding.

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A Extensions

Two extensions of our basic setting are particularly relevant for thinking about what kind of institutional settings may be desirable (in part to counteract the distortions associated with the hold-up problem discussed): one where some workers are not unionized, and one where employers also act as an entity. We study partial unionization in Section A.1 and collective bargaining in Section A.2. In this analysis we maintain the assumption of an egalitarian wage policy with no commitment, leading to a departure from efficient hiring.

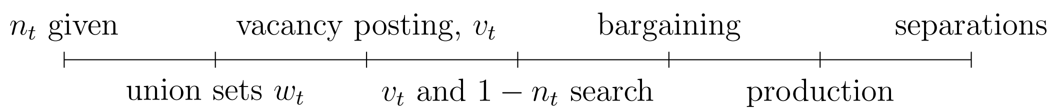
A.1 Partial unionization

In most countries, workers are free to decide on becoming union members. The analysis above does not allow for such a choice. One difficulty in incorporating endogenous membership into the model has to do with the formulation of the union's objective function. How does membership evolve over time, and how does the endogeneity of the unionization rate affect the incentives of the union when setting wages? The evolution of the unionization rate over time is a focus of some recent work, e.g., Dinlersoz and Greenwood (2012). Here, we stay short of a full analysis but nevertheless examine how key labor-market variables depend on the unionization rate. This analysis offers some preliminary insights into the welfare consequences of policies such as forbidding unions or requiring universal coverage of union wages.

In the analysis below, we use α to denote the fraction of workers who belong to the union. We treat α as exogenous, and assume that a worker's membership status is constant over time. We assume that the union's objective is to maximize the utility of its members. We also confine attention to steady states. In general, union workers may or may not earn higher wages than non-union workers. A particularly interesting steady state is a case which would make workers indifferent between being unionized and not, because this steady state can be interpreted as allowing workers to choose whether or not to become union members. As we will show, such a steady state exists for some parameter values.

Of course, we need to make clear how wages are determined for non-union workers. It is most natural here to simply adopt the standard assumption in the literature, i.e., one of decentralized Nash bargaining. We also need to make an assumption about whether the labor market is segmented by worker type—union vs. non-union—since firms in general are not indifferent about whom to meet. Our assumption is that the worker's union status is not observable ex ante so that matching is undirected. Moreover, we assume firms cannot discriminate based on union status later on either, with an identical separation probability for union and non-union workers.

Suppose, then, that only a share α of workers are unionized, while the rest bargain their wages bilaterally with firms. Both workers search in the same labor market, and firms learn the union status of workers only upon matching. At that time, bilateral bargaining occurs.



In this labor market, the union recognizes the presence of the non-union workers when deciding on its wage demands.

A.1.1 Analytical characterization

Beginning with a one-period example to gain intuition, we have that the union maximizes utility per member:

$$\underbrace{\big(n + \mu(\theta)(1-n)\big)}_{\text{employed}} w + \underbrace{(1-n)\big(1-\mu(\theta)\big)}_{\text{unemployed}} b,$$

by choice of w and θ, subject to the zero-profit condition: $\kappa = q(\theta)[\alpha(z-w)+(1-\alpha)(1-\gamma)S]$. Here $S \equiv z - b$ is the total surplus from a match between a firm and a non-union worker, and γ the worker's bargaining power, leaving the firm with share $1-\gamma$ of the surplus.

To see how the analysis compares to that with full unionization, we again use the zero-profit condition to substitute out the wage, as $w = z - \frac{\kappa}{\alpha q(\theta)} + \frac{1-\alpha}{\alpha}(1-\gamma)S$, in the union objective.

This leads to a maximization problem in θ only:

$$\max_{\theta} \big(\underbrace{n + \mu(\theta)(1-n)}_{\text{employed}}\big)\big(z - \frac{\kappa}{\alpha q(\theta)} + \frac{1-\alpha}{\alpha}(1-\gamma)S\big) + \underbrace{(1-n)\big(1-\mu(\theta)\big)}_{\text{unemployed}} b,$$

also taking as given n. As before, increasing θ increases employment, but at the cost of the lost wage income on new and existing workers required to raise θ. The expression differs from the one before for two reasons. First, the union wage now has a more limited impact on vacancy-creation, because of the non-union workers among the pool of unemployed. Raising θ through the union wage thus requires giving up more wage income: $\frac{\kappa}{\alpha q(\theta)}$ is greater with $\alpha < 1$. This works to reduce θ and raise the union wage, compared to the fully unionized case. Second, the tradeoff between the union wage and θ also depends on the firms' surplus from matching with non-union workers, $(1-\gamma)S = (1-\gamma)(z-b)$, in proportion with their prevalence among the unemployed, $(1-\alpha)/\alpha$. If the non-union firm surplus is large (relative to the union firm surplus), the union can target a higher θ without giving up as much in wages, which works to raise both θ and the union wage. In the next section, we illustrate how these effects manifest themselves in labor market outcomes.

In a fully dynamic setting, non-union workers and firms operate according to the usual Bellman equations:

$$\begin{aligned}
U_t &= \mu(\theta_t)E_t + (1-\mu(\theta_t))(b + \beta U_{t+1}), \\
E_t &= \hat{w}_t + \beta\delta U_{t+1} + \beta(1-\delta)E_{t+1}, \\
J_t &= z - \hat{w}_t + \beta(1-\delta)J_{t+1},
\end{aligned}$$

where U_t is the value of an unemployed worker, E_t the value of an employed worker, J_t the value of a filled job, and $\hat{w}_t$ the wage of a non-union worker. Based on these equations, the non-union worker-firm match surplus, defined as $S_t = E_t + J_t - b - \beta U_{t+1}$, satisfies

$$S_t = z - b + \beta(1-\delta)(1-\mu(\theta_{t+1})\gamma)S_{t+1},$$

where bilateral wage bargains imply $J_t = (1-\gamma)S_t$, and $E_t - b - \beta U_{t+1} = \gamma S_t$.

The zero-profit condition reads

$$\kappa = q(\theta_t)[\alpha(\sum_{s=0}^{\infty} \beta^s (1-\delta)^s z - W_t) + (1-\alpha)(1-\gamma)S_t].$$

Firms realize that union workers require a present value of wages of W_t, while non-union workers yield the firm a present discounted value of profits of $(1-\gamma)S_t$.

Using the zero-profit condition to substitute out wages in the union objective yields

$$\sum_{t=0}^{\infty} \beta^t [(n_t + \mu(\theta_t)(1-n_t))z + (1-\mu(\theta_t))(1-n_t)b - \theta_t(1-n_t)\frac{\kappa}{\alpha} + \frac{1-\alpha}{\alpha}(1-\gamma)\mu(\theta_t)(1-n_t)S_t]$$
$$- \frac{n_0 \kappa}{\alpha q(\theta_0)} + \frac{1-\alpha}{\alpha}(1-\gamma)n_0 S_0.$$

The objective here is the dynamic extension of the objective in the one-period example above, with terms slightly reordered. Also, S_t is of course not exogenous here (it was equal to $z-b$ in the one-period model) because it depends on future surpluses as well.

We consider the case of no commitment again and extend the Markov-perfect equilibrium definition to cover a general value of α. The equilibrium will have the functions $\Theta(n)$, for market tightness, $\tilde{V}(n)$, for the indirect utility of union members, and $S(n)$, the total surplus of the match between a firm and a non-union worker. Note that no new state variable is needed. The functions satisfy the following functional equations:

$$S(n) = z - b + \beta(1-\delta)(1-\mu(\Theta(n))\gamma)S(N(n,\Theta(n))),$$

$$\begin{aligned}\tilde{V}(n) = &(n + \mu(\Theta(n))(1-n))z + (1-\mu(\Theta(n)))(1-n)b - \Theta(n)(1-n)\frac{\kappa}{\alpha} \\ &+ \frac{1-\alpha}{\alpha}(1-\gamma)\mu(\Theta(n))(1-n)S(N(n,\Theta(n))) + \beta\tilde{V}(N(n,\Theta(n))),\end{aligned}$$

$$\begin{aligned}\Theta(n) = \arg\max_{\theta} &(n + \mu(\theta)(1-n))z + (1-\mu(\theta))(1-n)b - \theta(1-n)\frac{\kappa}{\alpha} - \frac{n\kappa}{\alpha q(\theta)} \\ &+ \frac{1-\alpha}{\alpha}(1-\gamma)(n + \mu(\theta)(1-n))S(N(n,\theta)) + \beta\tilde{V}(N(n,\theta)).\end{aligned}$$

It is straightforward to derive the functional first-order condition for the union here, but it is more complex since it contains both $S(N(n, \theta))$ and $S'(N(n, \theta))$, which cannot be eliminated with simple substitution. We therefore proceed directly to the quantitative analysis.

A.1.2 Quantitative results

We calibrate as in the benchmark case and vary α and γ to illustrate the workings of the model. The numerical analysis uses the same methods as above, with the mere difference that there is an additional unknown function S.[26]

Figure 3 plots the market tightness, wages, unemployment, and output as a function of the unionization rate α when worker bargaining power γ is relatively low.[27] In this case, firms pay non-union workers lower wages than union workers, making non-union workers more profitable to firms. The figure contrasts outcomes with the model calibrated to a monthly, versus annual, frequency. The annual horizon, on the right, illustrates the first mechanism discussed: higher unionization increases employment and output, as the union internalizes the effects of its wage demands on the labor market. In this case greater unionization brings the economy closer to efficiency.

With a monthly horizon, as depicted on the left, the union's commitment problem is more severe (reflected in lower vacancy creation than on the right). Here the second mechanism discussed becomes dominant for vacancy creation: the presence of non-union workers in the pool of unemployed helps mitigate the adverse effects of the commitment problem. As a result, greater unionization reduces employment and output, taking the economy farther away from efficiency.

Figure 4 turns to the case where workers are strong bargainers on their own. Interestingly, the figure shows that there is a level of unionization such that workers earn the same wages whether unionized or not. That is, if given the choice between becoming a union member or not, they would be indifferent. This steady state can be interpreted as the equilibrium outcome when workers can choose, at time zero, whether or not to be unionized.

[26] For details, see Appendix C.

[27] The specific value is 0.7; qualitatively, the graphs do not change if γ is lowered further.

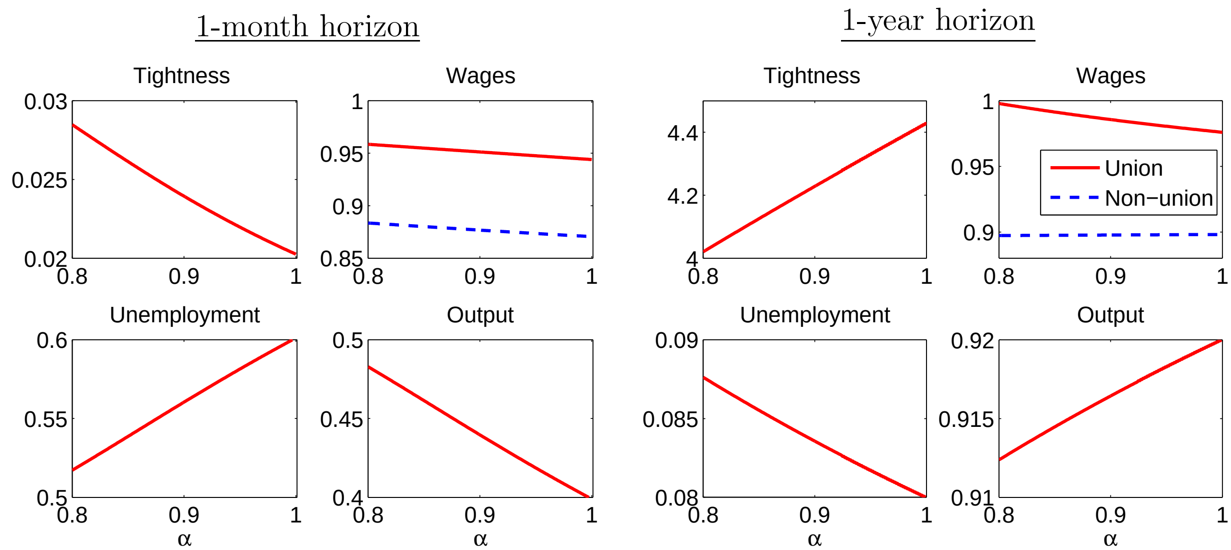


Figure 3: Labor markets when non-members are weak bargainers
Notes: The figure plots outcomes as a function of the unionization rate α for $\gamma = 0.7$.

This example demonstrates that *requiring* all workers to be unionized—or covered by the union wage—can be welfare improving. Interpreting the intermediate value of α where wages are the same for union and non-union workers as an equilibrium where workers can choose membership status: for that value of α, forced union membership would lead to better outcomes (and outlawing unions to worse), in a steady-state sense. In the previous example the situation is of course the reverse. There, workers would all choose to become unionized, leading to $\alpha = 1$, but outlawing unions could still be a good idea in the presence of large enough union distortions.

A.2 Collective bargaining

We can generalize the monopoly union framework to collective bargaining between a labor union and an employers' association, using a "right-to-manage" approach. Right-to-manage refers to firms having the right to decide on hiring independently, taking as given wages that are centrally bargained between the labor union and the employers' association. In the Mortensen-Pissarides framework this translates to hiring being determined by the usual zero-profit condition, given centrally bargained wages. Proceeding directly to the fully dynamic model, we adopt the same union objective in equation (3), and assume the employers'

1-year horizon

Figure 4: Labor markets when non-members are strong bargainers

Notes: The figure plots outcomes as a function of the unionization rate α for $\gamma = 0.95$.

association maximizes the present value of profits accruing to firms in equation (12). We look at both (joint) commitment and lack thereof.

A.2.1 With commitment

We now denote the bargaining power of the labor union vis a vis the employers' association by γ. With commitment to future wages, the collective bargaining problem solves the problem

$$\max_{\{w_t,\theta_t\}_{t=0}^{\infty}} \{\sum_{t=0}^{\infty} \beta^t \big[\big(n_t + \mu(\theta_t)(1-n_t)\big)w_t + (1-n_t)(1-\mu(\theta_t))b\big]\}^{\gamma}\{n_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t [z - w_t]\}^{1-\gamma}$$

subject to the law of motion (1) and the zero-profit condition (2). Note that, as before, the zero-profit condition implies that the employers' association objective reduces to representing initial matches only.[28]

To simplify, this bargaining problem can then be rewritten as a choice of a sequence of θ_t's.

[28] One could go into more detail in specifying alternative threat points in this bargaining problem, but we refrain to simply outlining the broader approach here.

Using the zero-profit condition, we arrive at

$$\max_{\{\theta_t\}_{t=0}^{\infty}} \{-\frac{n_0\kappa}{q(\theta_0)} + \sum_{t=0}^{\infty}\beta^t\big[\big(n_t+\mu(\theta_t)(1-n_t)\big)z+(1-n_t)(1-\mu(\theta_t))b-\theta_t(1-n_t)\kappa\big]\}^{\gamma}\{\frac{n_0\kappa}{q(\theta_0)}\}^{1-\gamma}$$

subject to the law of motion (1).

For thinking about how the solution differs from the monopoly union case, it is useful to note that future values of θ_t only enter the union objective, not the employers' association objective. Given this, one could equally well follow the earlier approach of reformulating the objective as

$$\{-\frac{n_0\kappa}{q(\theta_0)} + \big(n_0+\mu(\theta_0)(1-n_0)\big)z+(1-n_0)\big(1-\mu(\theta_0)\big)b-\theta_0(1-n_0)\kappa+\beta V\big(\,\cdot\,\big)\}^{\gamma}\{\frac{n_0\kappa}{q(\theta_0)}\}^{1-\gamma}$$

where $V(n)$ solves the recursive form of the planner's problem in equation (7). The solution to this planner's problem has θ constant at the efficient level, with $V(n)$ linear and increasing in n. The bargaining problem gives a different θ_0 in the initial period, however, depending on the bargaining power of the union vis-à-vis the employers' association. The employers' association moderates union wage demands, which translates into increased hiring. In fact, one can show that as union power γ declines, θ_0 increases from the monopoly union level.

Proposition 3. *If the labor union and the employers' association are able to commit to future wages, hiring is efficient after the initial period. In the initial period, hiring is efficient if* $n_0 = 0$*. If* $n_0 > 0$*, hiring is decreasing in the union bargaining power* γ*, and generically inefficient.*

A.2.2 Without commitment

We can adapt the right-to-manage formulation to the case of no commitment to future wages as follows. As before, we have an accounting equation for the continuation value

$$\tilde{V}(n) = \big(n+\mu(\Theta(n))(1-n)\big)z+(1-n)(1-\mu(\Theta(n)))b-\Theta(n)(1-n)\kappa+\beta\tilde{V}\big(N(n,\Theta(n))\big),$$

where

$$\Theta(n) := \arg\max_{\theta}\{-\frac{n\kappa}{q(\theta)} + \big(n+\mu(\theta)(1-n)\big)z + (1-n)(1-\mu(\theta))b \\ - \theta(1-n)\kappa + \beta\tilde{V}\big((1-\delta)(n+\mu(\theta)(1-n))\big)\}^{\gamma}\{\frac{n\kappa}{q(\theta)}\}^{1-\gamma}.$$

This differs from the monopoly union case only in that the choice of $\Theta(n)$ is now determined based on the bargaining problem instead of maximizing the union objective alone.

We proceed immediately to a numerical illustration, computed as in Section A.1. Figure 5 plots the outcomes for key labor-market variables as a function of the union bargaining power γ, over a range where steady-state unemployment takes on values both above and below the efficient level. As the the figure shows, the stronger is union bargaining power, the higher union wages, leading to higher unemployment and lower output. Moreover, the collective bargaining outcome is efficient for an intermediate value of γ.

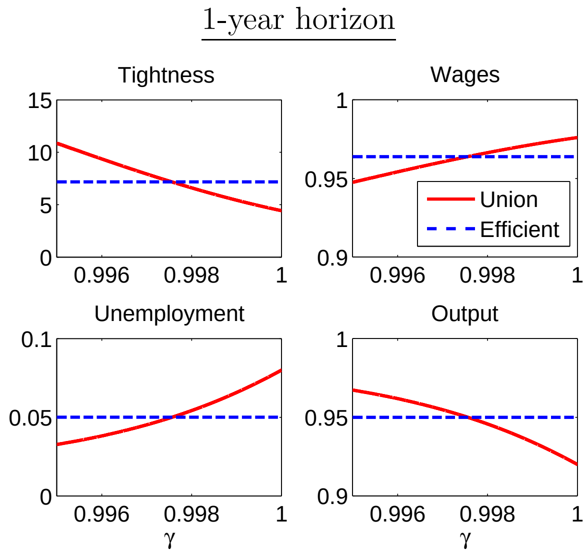


Figure 5: Labor market with collective bargaining

Notes: The figure plots outcomes as a function of the labor union bargaining power γ.

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B Proofs

Proof of relationship between union and planner objectives For the benchmark model with egalitarian wages, we need to show that

$$\sum_{t=0}^{\infty} \beta^t \big(n_t + h_t\big) w_t = \sum_{t=0}^{\infty} \beta^t \big[\big(n_t + h_t\big) z - \theta_t (1 - n_t)\kappa\big] - \frac{n_0 \kappa}{q(\theta_0)}, \tag{20}$$

where h_t stands for newly hired workers, i.e., $h_t = \mu(\theta_t)(1 - n_t)$.

First, note that the law of motion for employment implies that $n_t = (1-\delta)^t n_0 + \sum_{k=0}^{t-1} (1 - \delta)^{t-k} h_k$, so we can write $n_t + h_t = (1-\delta)^t n_0 + \sum_{k=0}^{t} (1-\delta)^{t-k} h_k$. Using this identity, the left hand side of equation (20) can then be written as

$$\sum_{t=0}^{\infty} \beta^t \big(n_t + h_t\big) w_t = n_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t w_t + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t} (1-\delta)^{t-k} h_k w_t. \tag{21}$$

The first term on the right of equation (21) can be written, using the zero-profit condition, as

$$-\frac{n_0 \kappa}{q(\theta_0)} + n_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t z.$$

The second term can be written, rearranging and using the zero-profit condition, as

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t} (1-\delta)^{t-k} h_k w_t &= \sum_{k=0}^{\infty} \beta^k h_k \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} w_t \\
&= -\sum_{k=0}^{\infty} \beta^k h_k \frac{\kappa}{q(\theta_k)} + \sum_{k=0}^{\infty} \beta^k h_k \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} z \\
&= -\sum_{t=0}^{\infty} \beta^t h_t \frac{\kappa}{q(\theta_t)} + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t} (1-\delta)^{t-k} h_k z.
\end{aligned}$$

These two terms combine into

$$\sum_{t=0}^{\infty} \beta^t[(n_t + h_t)z - \theta_t(1 - n_t)\kappa] - \frac{n_0\kappa}{q(\theta_0)}$$

i.e., the right hand side of equation (20). To see this, note that $h_t/q(\theta_t) = \theta_t(1 - n_t)$, and

$$n_0 \sum_{t=0}^{\infty} \beta^t(1-\delta)^t z + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t} (1-\delta)^{t-k} h_k z = \sum_{t=0}^{\infty} \beta^t(n_t + h_t)z.$$

With partial unionization, the zero-profit condition changes, affecting this derivation. The zero-profit condition now implies that the present value of wages W_t satisfy

$$W_t = \sum_{k=0}^{\infty} \beta^k(1-\delta)^k z - \frac{\kappa}{\alpha q(\theta_t)} + \frac{1-\alpha}{\alpha}(1-\gamma)S_t.$$

Using this new zero-profit condition, the first and second terms on the right of equation (21) can be written, respectively, as

$$-\frac{n_0\kappa}{\alpha q(\theta_0)} + n_0 \sum_{t=0}^{\infty} \beta^t(1-\delta)^t z + n_0\frac{1-\alpha}{\alpha}(1-\gamma)S_0,$$

and

$$-\sum_{t=0}^{\infty} \beta^t h_t \frac{\kappa}{\alpha q(\theta_t)} + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t} (1-\delta)^{t-k} h_k z + \frac{1-\alpha}{\alpha}(1-\gamma) \sum_{t=0}^{\infty} \beta^t h_t S_t.$$

These terms now combine into

$$\sum_{t=0}^{\infty} \beta^t[(n_t + h_t)z - \theta_t(1-n_t)\frac{\kappa}{\alpha} + \frac{1-\alpha}{\alpha}(1-\gamma)h_t S_t] - \frac{n_0\kappa}{\alpha q(\theta_0)} + \frac{1-\alpha}{\alpha}(1-\gamma)n_0 S_0.$$

□

Proof of Proposition 1 The union objective can be written in terms of the planner's value function, as in equation (3). The planner problem is standard, and known to have a linear solution $V(n)$, with the planner's choice of θ constant, independent of n. The union objective differs in the initial period by the $-n_0\kappa/q(\theta_0)$ term, however, which implies that the initial θ_0 is below the planner's choice, and this difference is greater the greater is n_0. $\square$

Proof of Proposition 2 Consider a steady state of the unionized economy, where $\Theta'(n) = -c$ for some $c > 0$. Using this fact, steady-state employment can be written as $n = (1 - \delta)\mu(\theta)/(1 - (1 - \delta)(1 - \mu(\theta)))$, equation (19) implies that the steady-state θ satisfies the equation

$$1 = \mu'(\theta)\frac{z - b}{\kappa} + \beta(1 - \delta)\big(1 - \mu(\theta) + \theta\mu'(\theta)\big) - \Delta(\theta), \tag{22}$$

where $\Delta(\theta) \equiv 0$ in the efficient outcome, and

$$\Delta(\theta) \equiv -\frac{1 - \delta}{\delta}\mu(\theta)\frac{q'(\theta)}{q(\theta)^2}[1 - \beta(1 - \delta)(1 - \mu(\theta) - \frac{\mu'(\theta)\delta c}{1 - (1 - \delta)(1 - \mu(\theta))})]$$

in the unionized economy. The term $\Delta(\theta)$ thus captures the union distortion. Under efficiency, the right-hand side of equation (22) is strictly decreasing in θ pinning down a unique steady-state θ (as long as $\mu'(0)\frac{z-b}{\kappa} + \beta(1 - \delta) > 1$).[29] Because the union distortion $\Delta(\theta)$ is strictly positive for any $\theta > 0$, the unionized economy must have lower steady-state θ. $\square$

Proof of Proposition 3 Similarly to Proposition 1, this result follows from writing the union problem in terms of the planner's value function, as in the text. $\square$

C Numerical approach

This section discusses our numerical solution approach in the case where the union cannot commit to future wages. We begin with the benchmark model in Section 3, and then turn to the extensions.

[29]Note that $m_u(v, u) = \mu(\theta) - \mu'(\theta)\theta$, an expression which is reasonable to assume to be increasing in θ.

C.1 Solving the benchmark model

As discussed in Section 4, solving the no-commitment union problem requires special care. With this in mind, we tried several different numerical approaches, comparing results across methods. We begin with an overview of the methods tried, before discussing the conclusions.

Local polynomial approximation approach to solving the generalized Euler equation Our baseline solution method is that outlined in Krusell, Kuruscu, and Smith (2002), based on solving the generalized Euler equation (19). This equation is a functional equation in $\Theta(n)$, defined over a range of values of n encompassing the steady state value of n. This approach amounts to calculating a Taylor polynomial approximating $\Theta(n)$ around its steady state. Calculating a k^{th} order polynomial involves first analytically differentiating the Euler equation k times with respect to n, acknowledging that $\Theta(n)$ is a function of n, and that $N(n, \Theta(n))$ is one as well. This yields $k + 1$ equations, which pin down the $k + 1$ coefficients in the polynomial. Evaluating the equations in steady state, with $n = \mu(\theta)(1-\delta)/(\delta+\mu(\theta)(1-\delta))$, the unknowns become the steady state values of $\theta, \theta', \theta'', ...$ up to the $k + 1$ derivative. Setting the last derivative to zero, the system determines these derivatives up to the k^{th} order. We first calculate the analytical derivatives, and the equations they yield, in Mathematica. We then turn to Matlab, solving for these derivatives (which determine the coefficients of the Taylor polynomial) using a non-linear equation solver. In practice, solving this system of equations can require a good initial guess, so we approach the problem iteratively, starting with a 0^{th} order Taylor polynomial and proceeding to successively higher-order polynomials, using the results from the previous step as initial guesses.

Global polynomial approximation approach to solving the generalized Euler equation As a functional equation, one can also look for a global solution to the Euler equation by approximating the solution $\Theta(n)$ with a cubic spline over some range of n's. Here we selected a grid on n, with the unknowns being the values of $\Theta(n)$ on that grid. These values determine the spline coefficients, which can be used to evaluate the Euler equation on the grid (and at intermediate points). This problem involves using a non-linear equation solver to find the values of $\Theta(n)$ on the grid, to minimize Euler equation errors.

Iterative approach to solving the generalized Euler equation One can also approach solving the Euler equation globally with an iterative approach. One way to do this is iterating backward, for example from a function $\Theta(n)$ which solves the final period optimization problem of a finite horizon union problem, with each step updating the values of $\Theta(n)$ on a fixed grid of n. In each step, for each grid point of n, we use the current set of $\Theta(n)$ to find n' next period, and then evaluate the right-hand-side of the Euler equation at these points using a cubic spline and the current set of $\Theta(n)$. One can then calculate a revised set of values of $\Theta(n)$, as the values of θ on the left-hand-side of the Euler equation.

Carroll's (2006) iterative approach to solving the generalized Euler equation One could also implement the iteration in the style of Carroll (2006), on an endogenous grid. Here we first rewrite the Euler equation with $N(n)$ as the unknown function instead of $\Theta(n)$. In doing so, the equation will have three successive values $\{n_{t-1}, n_t, n_{t+1}\}$, instead of the two $\{\theta_t, \theta_{t+1}\}$. At each iteration, we have for a grid of n_t, and corresponding values of $n_{t+1} = N_t(n_t)$. With these we can use the Euler equation to calculate the corresponding values of n_{t-1}. This gives a new grid on n_{t-1}, over which we have corresponding values $n_t = N_{t-1}(n_{t-1})$.

Value function iteration Finally, one can also use a value function iteration approach. Starting from a guess for $\tilde{V}$, at each step we first solve the maximization problem determining the optimal $\Theta(n)$ on a grid of n, and then calculate the preceding period's value of $\tilde{V}$ using the recursive equation. A natural starting point is a value of $\tilde{V}$ consistent with the final period of a finite-horizon problem. Here the recursive equation is not a contraction, however, so there is no guarantee of convergence.

Conclusions Each of these methods shows convergence toward very similar results, which is reassuring. In particular, they deliver functions $\Theta(n)$ which are quite similar. Moreover, the steady states we find are all stable. But each method also exhibits signs of numerical instability. To some extent we would anticipate this, because the recursive expressions need not be contractionary, and therefore the iterations may not converge from arbitrary initial guesses. Moreover, even the non-iterative approach to solving the Euler equation may be sensitive to numerical error. It is possible that these numerical issues are related to the presence of multiple equilibria, which confuse the algorithms. The fact that these varied numerical methods nevertheless show signs of convergence to very similar outcomes supports the idea that the equilibrium we study exists and is the relevant one to study. The infinite-horizon model using the concept of a differentiable Markov-perfect equilibrium thus delivers very similar intuition to the one-period example we started with, supporting it as the natural candidate to consider.

C.2 Solving the extended models

In addition to the basic non-stochastic no-commitment union problem discussed above, we also consider extensions to allow: i) aggregate shocks, ii) partial unionization, and iii) collective bargaining. We describe below how we extended our numerical methods in order to compute solutions in these cases as well.

Aggregate shocks Our baseline solution method can be extended to allow aggregate shocks by treating $\Theta(n, z)$ as a function of z also. We approximate this function again as a k^{th} order polynomial in n, but include also a linear, and quadratic term in z, as well as an interaction term. The coefficients of the polynomial in n are the same as in the non-stochastic case. Finding the terms involving z requires differentiating the generalized Euler equation with respect to z and proceeding with the same approach as described for n above. (It is important not to stop at just a linear term in z here, as the coefficient on z sharpens as more terms are added.)

To evaluate this procedure, we compare the results in the case of a fully persistent shock to the transitional dynamics to a permanent shock calculated using our baseline approach for non-stochastic problems.

Partial unionization and collective bargaining We extend our baseline solution method to these cases. There is no generalized Euler equation here, so we need to alter the approach somewhat. For simplicity, we describe how we do this in the context of the collective bargaining problem, which is slightly more straightforward.

In the collective bargaining problem, the first order condition involves $\tilde{V}'(\cdot)$, as before, but now also $\tilde{V}(\cdot)$, which prevents us from simply eliminating these functions to arrive at a generalized Euler equation. We can still implement the basic approach by allowing these $\tilde{V}(\cdot)$ to remain in the first order condition as we successively differentiate it k times (analytically). We simply need to use the recursive equation for $\tilde{V}(\cdot)$ to compute the successively higher order derivatives of $\tilde{V}(\cdot)$ which will show up in these $k+1$ equations. As before, in doing so we acknowledge the law of motion $N(n, \Theta(n))$ as we proceed with taking derivatives.

In the partial unionization problem, the approach is similar, but in addition to needing to calculate derivatives of $\tilde{V}(\cdot)$ based on the recursive equation for $\tilde{V}(\cdot)$, one also needs to calculate derivatives of $S(\cdot)$ based on the recursive equation for $S(\cdot)$.

To evaluate this procedure, we compare the results in these extensions with our baseline model in the special cases where, in the case of collective bargaining, the union has full bargaining power, and in the case of partial unionization, the unionization rate is one.