Competitive Search and Competitive Equilibrium

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Two Organizing Thoughts

- little model
- intuition

Foundations of Competitive Equilibrium

- how are prices formed without a Walrasian auctioneer?
  - fundamental question in search theory
- approach taken here: rethink what competitive equilibrium means
  - illustrate with a simple model
  - show usefulness through several extensions
    - search frictions and intermediation
    - risk aversion and inefficiency
    - heterogeneous assets and private information

Little Model

- two periods
- unit measure of risk-neutral consumers
  - nonnegative consumption of “fruit”
  - endowed with a single “tree” that has dividend δ
  - heterogeneous discount factors β, distribution G with density g
- trade trees for fruit

Competitive Equilibrium

Competitive Equilibrium

- individuals choose consumption and savings to maximize their utility

$$\max_{c,k'} c + \beta\delta k'$$

subject to the budget constraint

$$c + pk' = \delta + p$$

and nonnegativity constraints $c, k' \geq 0$, taking the price p as given

- denote the solution to this problem as $C(\beta; p)$
- markets clear: $\int C(\beta; p) g(\beta)\, d\beta = \delta$

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- markets clear: $\int C(\beta; p) g(\beta)\, d\beta = \delta$
- how does market achieve the equilibrium price p^*?

Alternative Approach

Concept

- individuals submit buy and sell schedules, $q_b(p;\beta)$ and $q_s(p;\beta)$
  - commitment to buy (sell) q_b (q_s) units at price p
- let $\Theta(p)$ be the buyer-seller ratio at p, $\Theta : [0,\infty) \mapsto [0,\infty]$
- there is rationing if $\Theta(p) \neq 1$:
  - sellers sell with probability $\min\{1, \Theta(p)\}$
  - buyers buy with probability $\min\{1, \Theta(p)^{-1}\}$
- can think of separate “markets” distinguished by p

Definition of Equilibrium I

- individuals choose demand and supply schedules:

$$\max_{q_b} \int (\beta\delta - p)\min\{1, \Theta(p)^{-1}\} q_b(p)\,dp + \max_{q_s} \int (p - \beta\delta)\min\{1, \Theta(p)\} q_s(p)\,dp$$

subject to the resource constraints

$$\int p q_b(p)\,dp \le \delta \text{ and } \int q_s(p)\,dp \le 1$$

taking as given $\Theta(p)$

- solution to this problem is $q_b(p;\beta)$ and $q_s(p;\beta)$
  - can solve the two problems separately

Definition of Equilibrium II

- compute measure of buyers and sellers at prices below p:

$$\mu_b(p) = \int_0^p \int q_b(p';\beta) g(\beta)\, d\beta\, dp'$$
$$\mu_s(p) = \int_0^p \int p' q_s(p';\beta) g(\beta)\, d\beta\, dp'$$

- "markets clear": $\Theta(p) = \dfrac{d\mu_b(p)}{d\mu_s(p)}$
  - no restriction on $\Theta(p)$ if $d\mu_b(p) = d\mu_s(p) = 0$

Equilibrium Characterization

- bang-bang solution
  - buy price $p_b(\beta) = \arg\max_p (\beta\delta - p)\min\{1, \Theta(p)^{-1}\}$
  - sell price $p_s(\beta) = \arg\max_p (p - \beta\delta)\min\{1, \Theta(p)\}$
- $\Theta(p) = \begin{cases} \infty \\ 1 \\ 0 \end{cases} \Leftrightarrow p \begin{matrix} < \\ = \\ > \end{matrix} p^*$
- combine: $\beta\delta \begin{matrix} > \\ = \\ < \end{matrix} p^* \Rightarrow \begin{cases} p_b(\beta) = p^* \\ p_b(\beta) \leq p^* \\ p_b(\beta) < p^* \end{cases}$ and $\begin{cases} p_s(\beta) > p^* \\ p_s(\beta) \geq p^* \\ p_s(\beta) = p^* \end{cases}$
- market clearing: $p^* G(p^*/\delta) = \delta(1 - G(p^*/\delta))$

Summary

- equilibrium allocation is competitive
- we can answer what happens if individuals try to trade at other prices
- we can extend the model in many directions
  - search frictions
  - risk aversion
  - indivisibilities
  - heterogeneous assets
  - private information

Search Frictions

Concept

- rationing occurs on both sides of the market:
  - sellers sell with probability $\pi_s(\Theta(p)) \leq \min\{1, \Theta(p)\}$
  - buyers buy with probability $\pi_b(\Theta(p)) \leq \min\{1, \Theta(p)^{-1}\}$
  - $\pi_s' > 0 > \pi_b'$ and $\pi_s(\theta) = \theta \pi_b(\theta)$
- individuals choose demand and supply schedules:

$$\max_{q_b} \int (\beta\delta - p)\pi_b(\Theta(p))q_b(p)dp + \max_{q_s} \int (p - \beta\delta)\pi_s(\Theta(p))q_s(p)dp$$

subject to the resource constraints

$$\int pq_b(p)dp \leq \delta \text{ and } \int q_s(p)dp \leq 1$$

- market clearing condition as before

Example

- three types: $\beta = \frac{1}{2}, 1, \frac{3}{2}$,
- $\pi_s(\theta) = \alpha\sqrt{\theta}$ (plus boundary conditions to ensure $\pi_s(\theta) \leq \min\{1, \theta\}$)
- suppose $\beta = 1$ does not trade
  - $\beta = \frac{1}{2}$ sells to $\beta = \frac{3}{2}$ at $p = 1$
- this cannot be an equilibrium:
  - $\beta = 1$ can profitable sell to $\beta = \frac{3}{2}$ at $1 + \varepsilon$
  - $\beta = 1$ can profitably buy from $\beta = \frac{1}{2}$ at $1 - \varepsilon$
  - $\beta = 1$ acts as an intermediary
    - if $g(1)$ is small, both intermediated and disintermediated trade
    - if $g(1)$ is large, all trade is intermediated

Risk Aversion

Risk Aversion

- preferences $\mathbb{E}u(c_1, c_2, \beta)$
- if individuals can avoid risk (insurance, law of large numbers):
  - $c_1 = \delta + \int p\Big(\pi_s(\Theta(p))q_s(p) - \pi_b(\Theta(p))q_b(p)\Big)dp$
  - $c_2 = \delta\left(1 + \int \Big(\pi_b(\Theta(p))q_b(p) - \pi_s(\Theta(p))q_s(p)\Big)\right)dp$
- similar to previous problem

Indivisibilities

- individuals must choose one buy price and one sell price
  - irrelevant with risk-neutrality
  - important with risk-aversion
- preferences $\mathbb{E}u(c_1, c_2, \beta)$

probability	c_1	c_2
$\pi_s(\Theta(p_s))\pi_b(\Theta(p_b))$	p_s	δ^2/p_b
$\pi_s(\Theta(p_s))(1-\pi_b(\Theta(p_b)))$	$\delta + p_s$	0
$(1-\pi_s(\Theta(p_s)))\pi_b(\Theta(p_b))$	0	$\delta + \delta^2/p_b$
$(1-\pi_s(\Theta(p_s)))(1-\pi_b(\Theta(p_b)))$	δ	δ

- incomplete markets skews towards safer behavior
  - reduction in the supply of intermediation, inefficiency

Heterogeneous Assets and Private Information

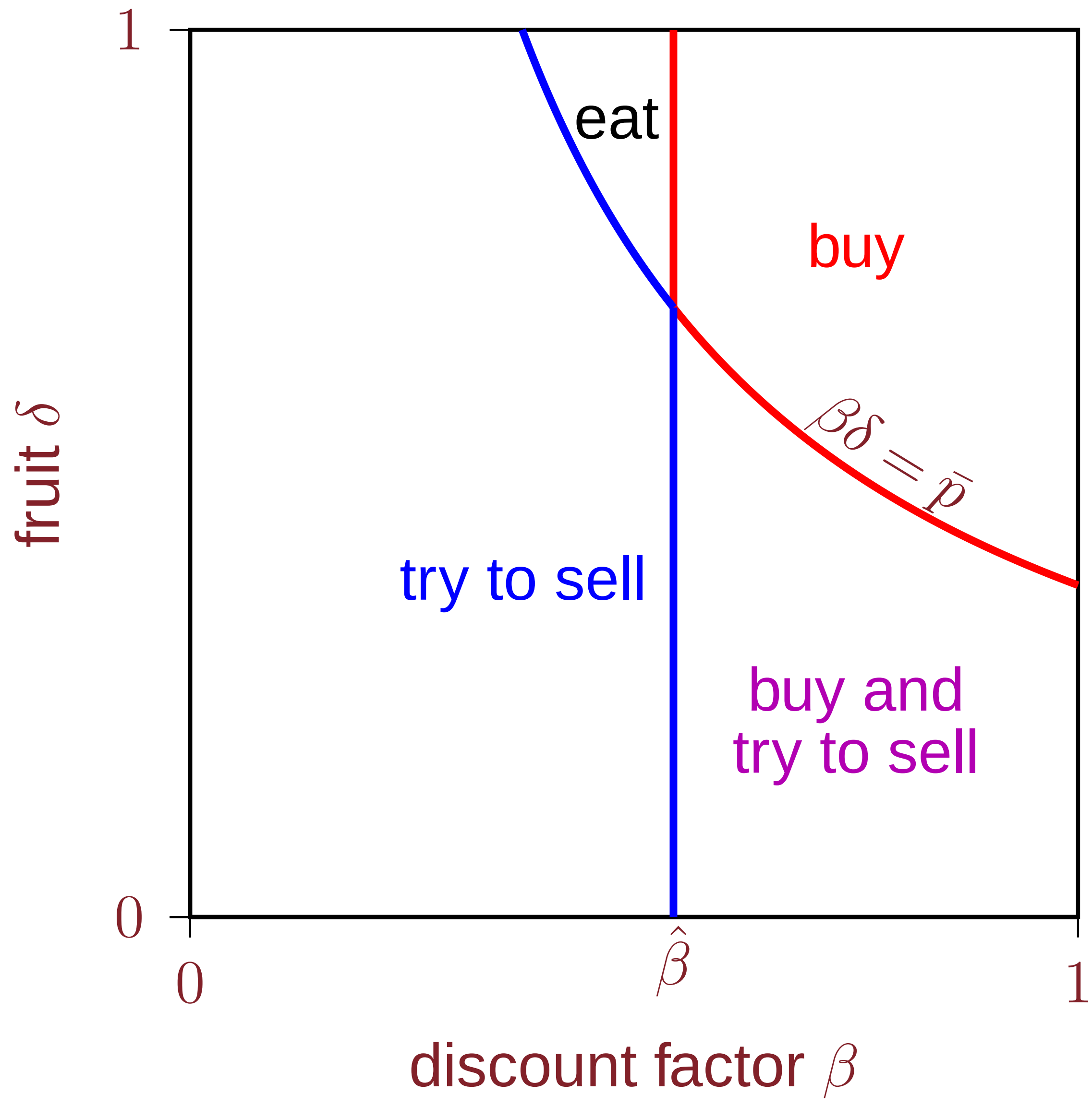
Heterogeneous Assets

- trees are heterogeneous in terms of δ
  - risk-neutrality and no search frictions for simplicity
  - joint distribution $G(\beta, \delta)$
- if δ is observable:
  - all assets have the same price-dividend ratio
  - nothing important changes

Private Information

- only the seller observes δ: must model buyers' beliefs
- β is observable:
  - separating equilibrium
  - patient individuals buy and impatient individuals attempt to sell
  - higher quality assets sell at higher price with lower probability
  - no "intermediation," i.e. simultaneous buying and selling
- β is unobservable:
  - semi-pooling equilibrium based on "continuation value" $\beta\delta$
  - patient individuals buy and impatient individuals attempt to sell
  - higher $\beta\delta$ sold at higher price with lower probability
  - "intermediation" by patient individuals with bad assets

Illustration



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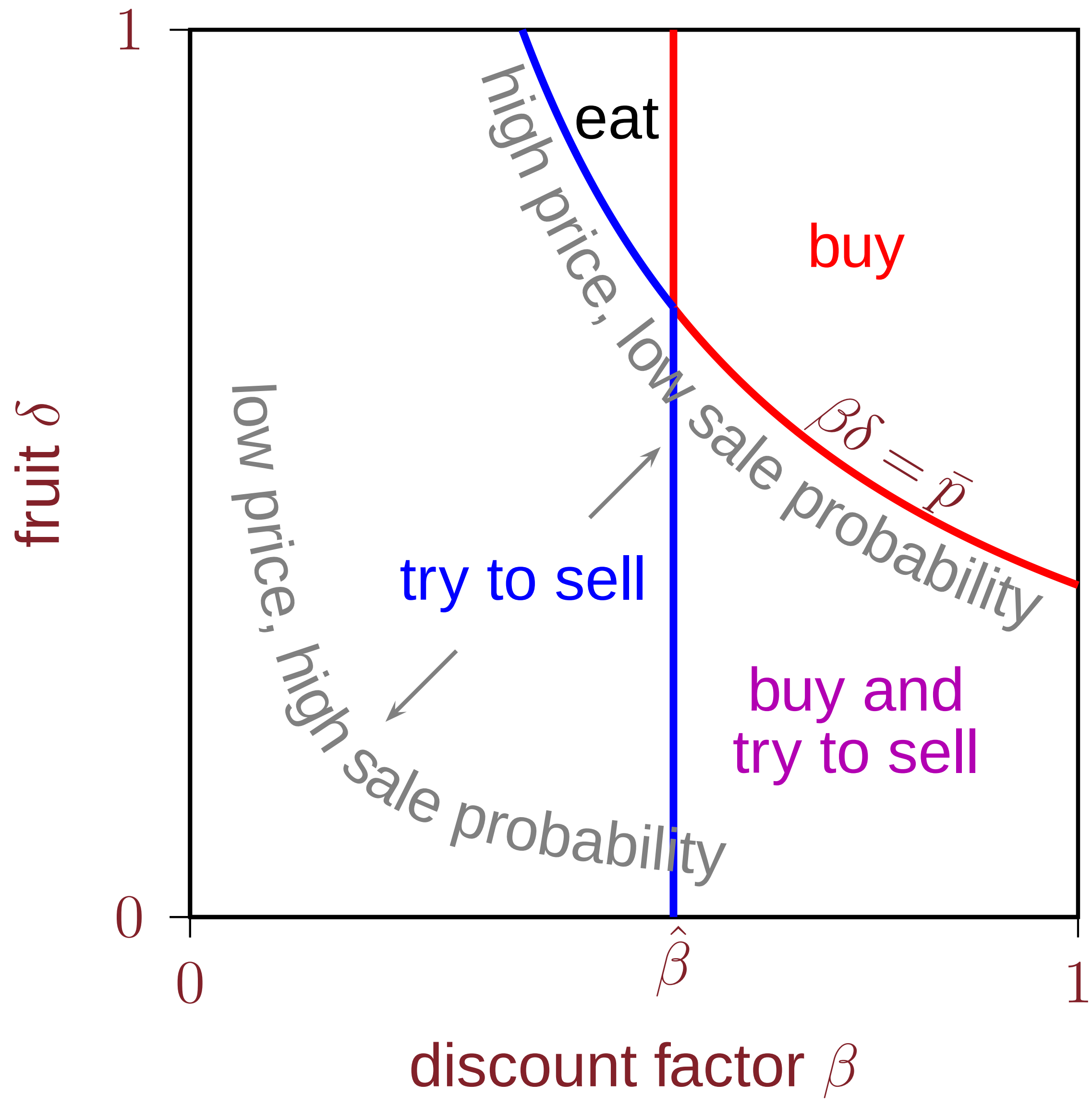

Conclusion

- competitive search equilibrium offers a flexible framework
- close link between search frictions and private information
  - similar notions of equilibrium
  - similar outcomes:
    - probabilistic trading
    - intermediation

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