Leverage Stacks and the Financial System

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Nemmers Conference
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30 April 2015
Leverage Stack:

entrepreneurs

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>Levered Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>13%</td>
</tr>
<tr>
<td>3%</td>
<td>12%</td>
</tr>
<tr>
<td>2%</td>
<td></td>
</tr>
</tbody>
</table>

At each level, borrower can pledge $\leq \frac{9}{10}$ of return $(\theta, \theta^*)$.
Entrepreneurial lending opportunities are i.i.d. (prob $\pi$)

e.g. five banks and $\pi = 2/5$:

Note: no mutual gross positions yet
To allow for mutual gross positions, suppose loans to entrepreneurs are long-term

down

every bank (even one of today’s non-lead banks) has some of these old assets on its b/sheet
– from when, in the past, it was a lead bank
<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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</thead>
<tbody>
<tr>
<td>capital investment</td>
<td>interbank bonds issued (short-term)</td>
</tr>
<tr>
<td>holdings (long-term)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>secured against</td>
</tr>
<tr>
<td>interbank bond holdings (short-term)</td>
<td>household bonds issued (short-term)</td>
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<tr>
<td></td>
<td>secured against</td>
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<tr>
<td></td>
<td>own equity</td>
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</table>
Should non-lead bank spend its marginal dollar on paying down (≡ not rolling over) old interbank debt secured against these old assets

⇒ return of 3%

or on buying new interbank debt @ 3%, levered by borrowing from households @ 2%

⇒ effective return of ≈ 12% ✓

That is, non-lead banks should “max out”
Hence there are mutual gross positions among non-lead banks:
Mutual gross positions among non-lead banks “certify” each others’ entrepreneurial loans and thus offer additional security to households

⇒ more funds flow in to the banking system, from households

⇒ more funds flow out of the banking system, to entrepreneurs

⇒ greater investment & aggregate activity

*But though the economy operates at a higher average level, it is susceptible to systemic failure*
MODEL

discrete time, dates \( t = 0, 1, 2, \ldots \)

at each date, single good (numeraire)

fixed set of agents (banks), who derive utility from their scale of investment

\( \Rightarrow \) a bank invests maximally if opportunity arises

in background: outside suppliers of funds (e.g. households)
Remove top of leverage stack:

replace this by "capital investment"

\[
\begin{cases}
\text{interest rate } r_t \\
\text{credit limit } \theta < 1
\end{cases}
\]

\[
\begin{cases}
\text{interest rate } r^* \\
\text{credit limit } \theta^* < 1
\end{cases}
\]
Capital investment

constant returns to scale; per unit of project:

\[
\begin{align*}
\text{date } t & \quad \text{date } t+1 & \quad \text{date } t+2 & \quad \text{date } t+3 & \quad \ldots \\
-1 & \quad a_{t+1} & \quad \lambda a_{t+2} & \quad \lambda^2 a_{t+3} & \quad \ldots
\end{align*}
\]

unit cost \quad \text{depreciation factor } \lambda < 1

where the economy-wide productivities \( \{a_{t+s}\} \) follow two-point i.i.d. process: \( a_{\text{high}} / a_{\text{low}} \)
Capital investment is illiquid: projects are specific to the investing bank.

However, the bank can issue “interbank bonds” (i.e. borrow from other banks) against its capital investment:

per unit of project, bank can issue

\[ \theta < 1 \text{ interbank bonds} \]

price path of interbank bonds: \( \{ q_t, q_{t+1}, q_{t+2}, \ldots \} \)
an interbank bond issued at date t-1 promises

\[ [ E_{t-1}a_t + \lambda E_{t-1}q_t ] \]
at date t

(i.e., bonds are short-term & creditor is promised (a fraction \( \theta \) of) expected project return next period + expected price of a new bond issued next period against residual flow of returns collateral securing old bond

\[ = \text{expected project return} + \text{expected sale price of new bond} \]
from the price path \( \{q_{t-1}, q_t, q_{t+1}, q_{t+2}, \ldots \} \) we can compute the interbank interest rates:

effective risk-free interbank interest rate, \( r_{t-1} \), between date \( t-1 \) and date \( t \) solves:

\[
q_{t-1} = \frac{1 - \delta_t}{1 + r_{t-1}} \left[ E_{t-1}a_t + \lambda E_{t-1}q_t \right]
\]

where \( \delta_t = \text{probability of default at date } t \) (endogenous)

NB in principle \( \delta_t \) is bank-specific
– but see Corollary to Proposition below
A bank can issue “household bonds” (i.e. borrow from households) against its holding of interbank bonds. Household bonds mimic interbank bonds:
– a household bond issued at date t-1 promises to pay \( [E_{t-1}a_t + \lambda E_{t-1}q_t] \) at date t per interbank bond, bank can issue

\[ \theta^* < 1 \] household bonds

at price

\[ q_{t-1}^* = \frac{1 - \delta_t}{1 + r^*} \left[ E_{t-1}a_t + \lambda E_{t-1}q_t \right] \]

households lend at \( r^* \)
These promised payments – on interbank and household bonds – are fixed at issue, date t-1, using that date’s expectation \( (E_{t-1}) \) of future returns & bond prices:

bonds are unconditional (no state-dependence)

In the event of, say, a fall in returns, or a fall in bond prices, the debtor bank must honour its fixed payment obligations, or risk default & bankruptcy

Assume bankruptcy \( \Rightarrow \) creditors receive nothing
A typical bank’s balance sheet at start of date $t$:

**Assets**
- Capital investment holdings ($k_t$)
- Interbank bond holdings ($b_t$)

**Liabilities**
- Interbank bonds issued ($\leq \theta k_t$)
- Household bonds issued ($\leq \theta b_t$)
- Own equity

The assets are secured against the liabilities.
lead bank’s flow-of-funds (assuming no default)

\[ i_t \leq a_t k_t - \left[ E_{t-1} a_t + \lambda E_{t-1} q_t \right] \theta k_t \]
- capital investment returns to other banks

\[ + \left[ E_{t-1} a_t + \lambda E_{t-1} q_t \right] b_t - \left[ E_{t-1} a_t + \lambda E_{t-1} q_t \right] \theta^* b_t \]
- payments from other banks payments to households

\[ + q_t \theta (\lambda k_t + i_t) \]
- sale of new interbank bonds

rollover
Hence, for a lead bank starting date \( t \) with \((k_t, b_t)\),

\[ b_{t+1} = 0 \]

and \( k_{t+1} = \lambda k_t + i_t \)

where \( i_t \) is given by

\[
(a_t - \theta E_{t-1} a_t) k_t + (1-\theta^*) [ E_{t-1} a_t + \lambda E_{t-1} q_t ] b_t + \theta(q_t - E_{t-1} q_t) \lambda k_t \]

\[ 1 - \theta q_t \]
non-lead bank’s flow-of-funds

\[ q_t b_{t+1} \leq a_t k_t - [E_{t-1}a_t + \lambda E_{t-1}q_t] \theta k_t \]

- purchase of other banks’ bonds
- returns
- payments to other banks

\[ + [E_{t-1}a_t + \lambda E_{t-1}q_t] b_t - [E_{t-1}a_t + \lambda E_{t-1}q_t] \theta^* b_t \]

- payments from other banks
- payments to households

\[ + q_t \theta \lambda k_t \]

- sale of new interbank bonds

\[ + q_t^* \theta^* b_{t+1} \]

- sale of new household bonds
Hence, for a non-lead bank starting date $t$ with $(k_t, b_t)$,

$$k_{t+1} = \lambda k_t$$

and $b_{t+1}$ is given by

$$q_t - \theta^* q_t^* + (1 - \theta^*) [E_{t-1} a_t + \lambda E_{t-1} q_t] b_t + \theta (q_t - E_{t-1} q_t) \lambda k_t$$

$$= q_t - \theta^* q_t^*$$
Net interbank bond holding = \( b_t - \theta k_t \)
each bank has its personal history of, at each past date, being either a lead or a non-lead bank

⇒ in principle we should keep track of how the distribution of \( \{k_t, b_t\} \)'s evolves (hard)

however, the great virtue of our expressions for \( k_{t+1} \) and \( b_{t+1} \) is that they are linear in \( k_t \) and \( b_t \)

⇒ aggregation is easy
At the start of date \( t \), let

\[
K_t = \text{banks' stock of capital investment}
\]

\[
B_t = \text{banks' stock of interbank bonds}
\]

\[
K_{t+1} = \lambda K_t + I_t \quad \text{where}
\]

\[
I_t = \text{banks' capital investment} = \pi \left\{ (a_t - \theta E_{t-1}a_t)K_t
\right.
\]

\[
\left. + (1-\theta^*)\left[ E_{t-1}a_t + \lambda E_{t-1}q_t \right]B_t
\right.
\]

\[
\left. + \theta(q_t - E_{t-1}q_t)\lambda K_t \right\}
\]

Investment is very sensitive to falls in the bond price.
and $B_{t+1}$ is given by

$$
(1-\pi) \left\{ (a_t - \theta E_{t-1}a_t)K_t \\
+ (1-\theta^*) \left[ E_{t-1}a_t + \lambda E_{t-1}q_t \right]B_t \\
+ \theta (q_t - E_{t-1}q_t)\lambda K_t \right\} \\
\frac{q_t - \theta^*q_t^*}{q_t - \theta^*q_t^*}
$$
Market clearing

Price $q_t$ clears the market for interbank bonds at each date $t$:

interbank banks’ bond demand $= B_{t+1}$

interbank banks’ bond supply $= \theta K_{t+1}$

Posit additional demand from “outside banks”:

$$D(r_t) = q_t \left( \theta K_{t+1} - B_{t+1} \right)$$

outside banks’ supply of loanable funds is increasing in risk-free interest rate $r_t$
The following results hold near to steady-state

Throughout, assume that most interbank loans come from the other inside banks, not from outside banks:

\[ q_t B_{t+1} \gg D(r_t) \]
As a preliminary, we need to confirm that non-lead banks will choose to lever their interbank lending with borrowing from households:

Lemma 1 \[ r_t > r^* \] iff

(A.1):

\[ \theta > \pi \theta \theta^* + (1-\pi)(1-\lambda + \lambda \theta) + (1-\pi)(1-\theta \theta^*)r^* \]
Lemma 2a

A fall in $a_t$ raises the current interest rate $r_t$

*Intuition:* $a_t \downarrow$ raises bond supply/demand ratio:

\[
\frac{\text{inside banks' bond supply}}{\text{inside banks' bond demand}} = \frac{\theta(\lambda K_t + \frac{\pi}{1-\theta q_t} W_t)}{\frac{1-\pi}{q_t-\theta^* q_t^*} W_t}
\]

where

\[
W_t = \left\{ \left( a_t - \theta E_{t-1} a_t \right) K_t + (1-\theta^*) \left[ E_{t-1} a_t + \lambda E_{t-1} q_t \right] B_t + \theta (q_t - E_{t-1} q_t) \lambda K_t \right\}
\]

which implies $r_t \uparrow$
Lemma 2b

For $s \geq 0$, a rise in $r_{t+s}$ raises $r_{t+s+1}$

Intuition: $r_{t+s} \uparrow \implies (1 + r_{t+s})D(r_{t+s}) \uparrow$

\[
\text{debt (inclusive of interest) owed by inside banks to outside banks at date } t+s+1
\]

\[\implies W_{t+s+1} \downarrow \text{ (debt overhang)}\]

\[\implies r_{t+s+1} \uparrow \text{ (cf. Lemma 2a)}\]
Lemma 2c

A rise in future interest rates raises the current interest rate if \((A.2)\): \(\theta^*\pi > (1 - \lambda + \lambda\pi)^2\)

*Intuition:* a rise in any of \(E_t r_{t+1}, E_t r_{t+2}, E_t r_{t+3}, \ldots\)

\[\Rightarrow E_t q_{t+1} \downarrow \Rightarrow q_t^* = \frac{1 - \delta_{t+1}}{1 + r^*} \left\{ E_t a_{t+1} + \lambda E_t q_{t+1} \right\} \downarrow\]

\[\Rightarrow \text{ratio of inside banks’ bond supply/demand}\]

\[= \frac{\theta \left( \lambda K_t + \frac{\pi}{1 - \theta q_t} W_t \right)}{\frac{1 - \pi}{q_t - \theta^* q_t} W_t} \uparrow \Rightarrow r_t \uparrow\]

under \((A.2)\), this channel dominates (borrowing from households \(\downarrow\))
amplification through interest rate cascades:

\[ a_t \downarrow \]

\[ r_t \uparrow \]

\[ r_t+1 \uparrow \]

\[ r_{t+2} \uparrow \]

\[ r_{t+3} \uparrow \]

\[ \Rightarrow q_t \downarrow \]

\[ \Rightarrow l_t \downarrow \downarrow \]
collateral-value multiplier:

- interbank bond prices ↓
- collateral values ↓
- borrowing from households ↓
- net interbank lending by non-lead banks ↓
- interbank interest rates ↑
broad intuition:

negative shock

⇒ interbank interest rates ↑ and bond prices ↓

⇒ banks’ household borrowing limits tighten

⇒ funds are taken from banking system, just as they are most needed
fall in interbank bond prices

⇒ banks may have difficulty rolling over their debt, and so be vulnerable to failure

“most vulnerable” banks:

banks that have just made maximal capital investment (because they hold no cushion of interbank bonds)

Failure of these banks can precipitate a failure of the entire banking system:
Proposition (systemic failure)

In addition to Assumption (A.1), assume

(A.3): \[ \theta^* > (1-\pi) \lambda \]

If the aggregate shock is enough to cause the most vulnerable banks to fail, then all banks fail (in the order of the ratio of their capital stock to their holding of other banks’ bonds).

NB In proving this Proposition, use is made of the steady-state (ergodic) distribution of the \( \{k_t, b_t\} \)’s across banks
Corollary

At each date t, the probability of default, \( \delta_t \), is the same for all inside banks.

We implicitly assumed this earlier – in effect, we have been using a guess-and-verify approach.

Banks make no attempt to self-insure – e.g. by lending to “less risky” banks (because there are none: all banks are equally risky).
Parameter consistency?

Assumptions (A.1), (A.2) and (A.3) are mutually consistent:

e.g. \( \pi = 0.1 \)

\( \lambda = 0.975 \)

\( \theta = \theta^* = 0.9 \)

\( r^* = 0.02 \)
key point: non-lead banks are both borrowers and lenders in the interbank market

new interbank borrowing at r (rollover)

non-lead bank

new interbank lending at r

new household borrowing at r*

secured against

notice multiplier effect: if for some reason bank’s value of new interbank borrowing ↓ (by x dollars, say)

⇒ bank’s value of new interbank lending ↓↓ (by >> x dollars, because of household leverage)

⇒ bank’s *net* interbank lending ↓
if the "household-leverage multiplier" exceeds the "leakage" to lead banks
then we get amplification along the chain