Income Dynamics and Consumption Inequality: Nonlinear Persistence and Partial Insurance

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Short Course, Northwestern University

[Updated papers and references on my web page]

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2. To explore *the nonlinear nature of income shocks and the implications for consumption dynamics and inequality*. 
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In particular, the aim is:

1. To consider *alternative ways of modelling persistence*, and
2. To explore *the nonlinear nature of income shocks and the implications for consumption dynamics and inequality*.

⇒ e.g. US Household Panel data and Norwegian Population Register data.
New data on consumption and family income sources

I. Administrative linked data: e.g. Norwegian population register.

- Linked registry databases with unique individual identifiers.
  - Containing records for **every Norwegian from 1967 to 2014**.
  - Detailed demographic and socioeconomic information (market income, cash transfers). Recent links to real estate and assets; and to hours of work. New consumption measurements.

- Family identifiers allow to match spouses and children.
  - see Blundell, Graber and Mogstad (2015).
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II. Newly designed panel surveys: e.g. PSID since 1999.

- Collection of consumption and assets had a major revision in 1999
  - ~70% of consumption expenditures, more since 2004.
  - The sum of food at home, food away from home, gasoline, health, transportation, utilities, clothing etc.

- Earnings and hours for all earners; Assets measured in each wave.
  - see Blundell, Pistaferri and Saporta-Eksten (2016).
A prototypical “canonical” panel data model of (log) family (earned) income $y_{it}$ is:

$$y_{it} = \eta_{it} + \epsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$  

where $y_{it}$ is net of a systematic component, $\eta_{it}$ is a random walk with innovation $v_{it}$,

$$\eta_{it} = \eta_{it-1} + v_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$  

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Consumption growth is then related to income shocks:

$$\Delta c_{it} = \phi_t v_{it} + \psi_t \epsilon_{it} + v_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$  

where $c_{it}$ is log total consumption net of a systematic component,

$\phi_t$ is the transmission of persistence shocks $v_{it}$, and

$\psi_t$ the transmission of transitory shocks;

- the $v_{it}$ are taste shocks, assumed to be independent across periods.
Covariance Restrictions

Baseline panel data model specification:

\[ \Delta c_{it} = \phi v_{it} + \psi \varepsilon_{it} + \nu_{it}, \]

\[ \Delta y_{it} = v_{it} + \Delta \varepsilon_{it}, \]

Imposing covariance restrictions:

\[ \text{var}(\Delta c_{it}) = \phi^2 \sigma_v^2 + \psi^2 \sigma_\varepsilon^2 \]

\[ \text{var}(\Delta y_{it}) = \sigma_\eta^2 + 2 \sigma_\varepsilon^2 \]

\[ \text{cov}(\Delta y_{it} \Delta y_{it-1}) = -\sigma_\varepsilon^2 \]

\[ \text{cov}(\Delta c_{it} \Delta y_{it}) = \phi \sigma_v^2 + \psi \sigma_\varepsilon^2 \]

\[ \text{cov}(\Delta c_{it-1} \Delta y_{it}) = \psi \sigma_\varepsilon^2 \]

For \( T \geq 3 \), BPP include time(age) variation in the \( \sigma^2 \) and insurance parameters,

BPP allow for measurement error and extend to MA(1) transitory shocks,

BP develop these covariance restrictions for repeated cross-sections.
Linking Income Dynamics to Consumption Inequality

More specifically, to account for the impact of income shocks on the evolution of consumption inequality we introduce *transmission* or *partial insurance* parameters, writing consumption growth as:

\[
\Delta \ln C_{it} \equiv \gamma_{it} + \Delta Z_{it} \varphi + \phi_t \nu_{it} + \psi_t \varepsilon_{it} + \xi_{it}
\]

\(\phi_t\) and \(\psi_t\) provide the link between the consumption and income distributions - \(\nu_{it}\) the permanent and \(\varepsilon_{it}\) the transitory shock to income.
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\[ \Delta \ln C_{it} \approx \gamma_{it} + \Delta Z_{it} \varphi + \phi_t v_{it} + \psi_t \varepsilon_{it} + \xi_{it} \]

\(\phi_t\) and \(\psi_t\) provide the link between the consumption and income distributions - \(v_{it}\) the permanent and \(\varepsilon_{it}\) the transitory shock to income.

- For a simple benchmark intertemporal consumption model for consumer of age \(t\), BLP (2013) show

\[ \phi_t = (1 - \pi_{it}) \quad \text{and} \quad \psi_t = (1 - \pi_{it}) \gamma_{Lt} \]

where

\[ \pi_{it} \approx \frac{\text{Assets}_{it}}{\text{Assets}_{it} + \text{Human Wealth}_{it}} \]

and \(\gamma_{Lt}\) is the annuity value of a temporary shock to income for an individual aged \(t\) retiring at age \(L\).

[Easily extend to ARMA processes for income.]
- This “standard” framework implies a set of extended covariance restrictions for panel data on consumption and income,

\( \Delta \) allowing the insurance parameters and variances to depend on age and education turns out to be key (analysis of PSID and Norwegian data).

\( \Rightarrow \) can show (over-)identification and efficient estimation via nonlinear GMM, see Blundell, Preston and Pistaferri (AER, 2008).

\( \Rightarrow \) Blundell, Pistaferri and Saporta (AER, 2016) - develop the nonlinear GMM panel data estimator for wage shocks and family labor supply.

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• Linearity of the income (or wage) process simplifies identification and estimation.

▷ However, by construction, it *rules out the nonlinear transmission of shocks.*
Motivation

• The aim in this lecture is to step back and take a different tack - develop an alternative approach to modeling persistence in which the impact of past shocks on current incomes/earnings can be altered by the size and sign of new shocks.

• This new framework draws on a flurry of recent work on nonlinearity and heterogeneity in the dynamics of inequality and income risk (full references in Arellano, Blundell and Bonhomme, 2017).

• The idea is to have a framework allows:
  ⇒ “unusual” shocks to wipe out the memory of past shocks, and
  ⇒ future persistence of a current shock to depend on the future shocks.
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• The idea is to have a framework allows:
  ⇒ “unusual” shocks to wipe out the memory of past shocks, and
  ⇒ future persistence of a current shock to depend on the future shocks.

• We will see that the presence of “unusual” shocks matches the data and has a key impact consumption and saving over the life cycle.
Background papers

- Blundell, Low and Preston [BLP] ‘Decomposing changes in income risk using consumption data’ (*QE*, 2013)

maybe finding time to look at family labor supply in:

-> on my website http://www.ucl.ac.uk/~uctp39a/pub.html
Nonlinear Persistence

- Consider a cohort of households, \( i = 1, \ldots, N \), and denote age as \( t \). Let \( y_{it} \) denote log-labor income, net of age dummies

\[
y_{it} = \eta_{it} + \varepsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.
\]

\( \eta_{it} \) follows a general first-order Markov process (can be generalised).

- Denoting the \( \tau \)th conditional quantile of \( \eta_{it} \) given \( \eta_{i,t-1} \) as \( Q_t(\eta_{i,t-1}, \tau) \), we specify

\[
\eta_{it} = Q_t(\eta_{i,t-1}, u_{it}), \quad \text{where } (u_{it} | \eta_{i,t-1}, \eta_{i,t-2}, \ldots) \sim \text{Uniform}(0, 1).
\]

\( \varepsilon_{it} \) has zero mean, independent over time.

\( \eta_{it} \) and the marginal distributions \( F_{\varepsilon_t} \) can all be age \( (t) \) specific.
A measure of nonlinear persistence

• This framework allows for nonlinear dynamics of income.

• To see this, consider the following measure of persistence

\[ \rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_t(\eta_{i,t-1}, \tau)}{\partial \eta}. \]

⇒ \( \rho_t(\eta_{i,t-1}, \tau) \) measures the persistence of \( \eta_{i,t-1} \) when, at age \( t \), it is hit by a shock \( u_{it} \) that has rank \( \tau \). Measures the persistence of histories.

▷ Allows a general form of conditional heteroscedasticity, skewness and kurtosis.
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▷ Allows a general form of conditional heteroscedasticity, skewness and kurtosis.

- In the “canonical model” \( \eta_{it} = \eta_{i,t-1} + v_{it} \), with \( v_{it} \) independent over time and independent of past \( \eta' \)s,

\[ \eta_{it} = \eta_{i,t-1} + F_{v_t}^{-1}(u_{it}) \quad \Rightarrow \quad \rho_t(\eta_{i,t-1}, \tau) = 1 \text{ for all } (\eta_{i,t-1}, \tau). \]
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– But what is the evidence for such nonlinearities in persistence?
Some motivating evidence: Quantile autoregressions of log-earnings

\[
\frac{\partial Q_{y_t|y_{t-1}}(y_i, t-1, \tau)}{\partial y}
\]


Estimates of the average derivative of the conditional quantile function of \( y_{it} \) given \( y_{i,t-1} \).
Conditional skewness, Norwegian administrative data

Family income

Individual income

Note: Skewness measured as a nonparametric estimate of

\[
\frac{Q_{y|y_{t-1}}(y_{i,t-1}, .9) - Q_{y|y_{t-1}}(y_{i,t-1}, .1)}{Q_{y|y_{t-1}}(y_{i,t-1}, .9) - Q_{y|y_{t-1}}(y_{i,t-1}, .1)} - 2Q_{y|y_{t-1}}(y_{i,t-1}, .5)
\]

Age 30-59, years 2005-2006.
Life-cycle model: illustrative simulation

- Calibration based on Kaplan and Violante [KV] (2010). Households enter the labor market at age 25, work until 60, and die with certainty at age 90.

- A single risk-free, one-period bond with return $1 + r$ ($r = .03$),

  $$A_t = (1 + r)A_{t-1} + Y_{t-1} - C_{t-1}.$$ 

- Log-earnings are $\ln Y_t = \kappa_t + \eta_t + \varepsilon_t$, where $\kappa_t$ is a deterministic age profile. In period $t$ agents know $\eta_t$, $\varepsilon_t$ and their past values, but not $\eta_{t+1}$ or $\varepsilon_{t+1}$ (no advance information).

- Period-$t$ optimization

  $$V_t(A_t, \eta_t, \varepsilon_t) = \max_{C_t} u(C_t) + \beta \mathbb{E}_t [V_{t+1}(A_{t+1}, \eta_{t+1}, \varepsilon_{t+1})],$$

  where $u(\cdot)$ is CRRA ($\gamma = 2$), and $\beta = 1/(1 + r) \approx .97$.

- We compare the results for the canonical earnings process used by KV, with our nonlinear process.
Simulation results

Consumption (age 37) by decile of $\eta_{t-1}$

Average consumption over the life-cycle

Note: Blue is nonlinear earnings process, Green is canonical earnings process.
An Empirical Consumption Rule

- Let \( c_{it} \) and \( a_{it} \) denote log-consumption and assets (beginning of period) net of age dummies.

- Our empirical specification is based on

\[
c_{it} = g_t \left( a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it} \right) \quad t = 1, ..., T,
\]

where \( \nu_{it} \) are independent across periods, and \( g_t \) is a nonlinear, age-dependent function, monotone in \( \nu_{it} \).

- \( \nu_{it} \) may be interpreted a taste shifter that increases marginal utility. We normalize its distribution to be standard uniform in each period.
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  - $\nu_{it}$ may be interpreted a taste shifter that increases marginal utility. We normalize its distribution to be standard uniform in each period.

> This consumption rule is consistent, in particular, with the standard life-cycle model on the earlier slide.

> Can allow for individual heterogeneity, advance information and habits.
Insurance coefficients

- With consumption specification given by

\[ c_{it} = g_t(a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it}), \quad t = 1, \ldots, T, \]

consumption responses to \( \eta \) and \( \varepsilon \) are

\[ \phi_t(a, \eta, \varepsilon) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta, \varepsilon, \nu)}{\partial \eta} \right], \quad \psi_t(a, \eta, \varepsilon) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta, \varepsilon, \nu)}{\partial \varepsilon} \right]. \]

\( \Phi_t(a, \eta, \varepsilon) \) and \( \Psi_t(a, \eta, \varepsilon) \) reflect the transmission of the persistent and transitory earnings components, respectively.
Insurance coefficients

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\( \triangleright \) \( \phi_t(a, \eta, \varepsilon) \) and \( \psi_t(a, \eta, \varepsilon) \) reflect the transmission of the persistent and transitory earnings components, respectively.

- The marginal effect of an earnings shock \( u \) on consumption is

\[ \mathbb{E}\left[ \frac{\partial}{\partial u} \bigg|_{u=\tau} g_t(a, Q_t(\eta, u), \varepsilon, \nu) \right] = \phi_t(a, Q_t(\eta, \tau), \varepsilon) \frac{\partial Q_t(\eta, \tau)}{\partial u}. \]
Earnings: identification

• For $T = 3$, Wilhelm (2012) gives conditions under which the distribution of $\varepsilon_{i2}$ is identified.

  – In particular, completeness of the pdfs of $(y_{i2}|y_{i1})$ and $(\eta_{i2}|y_{i1})$. This requires $\eta_{i1}$ and $\eta_{i2}$ to be dependent.

• In this research we use this result to establish identification of the earnings model.

• Apply the result to each of the three-year sub-panels $t \in \{1, 2, 3\}$ to $t \in \{T - 2, T - 1, T\}$

⇒ The marginal distribution of $\varepsilon_{it}$ are identified for $t \in \{2, 3, \ldots, T - 1\}$.
⇒ By independence the joint distribution of $(\varepsilon_{i2}, \varepsilon_{i3}, \ldots, \varepsilon_{i,T-1})$ is identified.
⇒ By deconvolution the distribution of $(\eta_{i2}, \eta_{i3}, \ldots, \eta_{i,T-1})$ is identified.

• The distribution of $\varepsilon_{i1}$, $\eta_{i1}$, and $\varepsilon_{iT}$, $\eta_{iT}$ are not identified in general.
Consumption: assumptions

• \( u_{it} \) and \( \varepsilon_{it} \) are independent of past earnings shocks and past asset holding, for \( t \geq 1 \), where \( \eta_{it} = Q_t(\eta_{i,t-1}, u_{it}) \).

• We let \( \eta_{i1} \) and \( a_{i1} \) be arbitrarily dependent;
  – this is important, because asset accumulation upon entry in the sample may be correlated with past persistent shocks.

• Denoting \( \eta_i^t = (\eta_{it}, \eta_{i,t-1}, \ldots, \eta_{i1}) \), we assume (in this talk) that:\n  \( a_{it} \) is independent of \( (\eta_i^{t-1}, a_i^{t-2}, \varepsilon_i^{t-2}) \) given \( (a_{i,t-1}, c_{i,t-1}, y_{i,t-1}) \);
  – consistent with the accumulation rule in the standard life-cycle model with one single risk-less asset.
Consumption: initial assets

Let \( y = (y_1, \ldots, y_T) \). We have

\[
f(a_1 | y) = \int f(a_1 | \eta_1, y) f(\eta_1 | y) d\eta_1
\]

\[
= \int f(a_1 | \eta_1) f(\eta_1 | y) d\eta_1,
\]

where we have used that \( u_{it} \) and \( \varepsilon_{it} \) are independent of \( a_{i1} \).

Note that \( f(\eta_1 | y) \) is identified from the earnings process alone.

If \( f(\eta_1 | y) \) is complete, then \( f(a_1 | \eta_1) \) is identified.

Structure is as in the NPIV problem where \( \eta_1 \) is the endogenous regressor and \( y \) is the instrument.
Consumption: first period

- We have

\[ f(c_1, a_1|y) \equiv \int f(c_1, a_1|\eta_1, y)f(\eta_1|y)d\eta_1 \]

and given our assumptions

\[ f(c_1, a_1|y) = \int f(c_1|a_1, \eta_1, y_1)f(a_1|\eta_1)f(\eta_1|y)d\eta_1. \]

- \( f(a_1|\eta_1) \) can be treated as known.

- Provided we have completeness in \( (y_2, ..., y_T) \) of \( f(\eta_1|y_1, y_2, ..., y_T) \), then \( f(c_1|a_1, \eta_1, y_1) \), is identified.

- Intuition: \( y_{i2}, ..., y_{iT} \) are used as “instruments” for \( \eta_{i1} \).

- Subsequent periods discussed in ABB (2017), briefly here...
Consumption: subsequent periods

• We have

\[ f(a_2|c_1, a_1, y) = \int f(a_2|c_1, a_1, \eta_1, y_1)f(\eta_1|c_1, a_1, y)\,d\eta_1 \]

\[ f(c_2|a_2, c_1, a_1, y) = \int f(c_2|a_2, \eta_2, y_2)f(\eta_2|a_2, c_1, a_1, y)\,d\eta_2. \]

• By induction it can be shown that the joint density of \( \eta \)'s, consumption, assets, and earnings is identified provided, for all \( t \geq 1 \), the distributions of \( (\eta_{it}|c_t^i, a_t^i, y_i) \) and \( (\eta_{it}|c_{t-1}^i, a_t^i, y_i) \) are complete in \( (c_{t-1}^i, a_{t-1}^i, y_{t-1}, y_{i,t+1}, \ldots, y_{iT}). \)

• Intuition: lagged consumption and assets, as well as lags and leads of earnings, are used as instruments for \( \eta_{it} \).
Similar techniques can be used in the presence of *advance information*, e.g.

\[ c_{it} = g_t \left( a_{it}, \eta_{it}, \eta_{i,t+1}, \varepsilon_{it}, \nu_{it} \right), \]

or *consumption habits*, e.g.

\[ c_{it} = g_t \left( c_{i,t-1}, a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it} \right). \]

▷ also cases where the consumption rule depends on lagged \( \eta \), or when \( \eta \) follows a second-order Markov process. (See Section 3 in *ABB*, 2017).
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Also cases where the consumption rule depends on lagged \( \eta \), or when \( \eta \) follows a second-order Markov process. (See Section 3 in ABB, 2017).

Households differ in their initial productivity \( \eta_1 \) and initial assets, the panel data provide opportunities to allow for additional, unobserved heterogeneity in earnings and consumption.

For example: heterogeneity \( \xi_i \) in discounting or preferences, or heterogeneity \( \tilde{\xi}_i \) in the Markovian transitions of \( \eta_{it} \).
• Consumption rule with \textit{unobserved heterogeneity}:

\[ c_{it} = g_t(a_{it}, \eta_{it}, \varepsilon_{it}, \zeta_i, \nu_{it}). \]

• We assume that \( u_{it} \) and \( \varepsilon_{it} \), for \( t \geq 1 \), are independent of \( (a_{i1}, \zeta_i) \).

• The distribution of \( (a_{i1}, \zeta_i, \eta_{i1}) \) is unrestricted.

• A combination of the above identification arguments and the main result of Hu and Schennach (08) identifies:

– the period-\( t \) consumption distribution \( f(c_t | a_t, \eta_t, y_t, \zeta) \), and

– the distribution of initial conditions \( f(\eta_1, \zeta, a_1) \).
Data: PSID

• (New) PSID 1999 - 2009, we use 6 waves (every other year), as in BPS.

• $C_{it}$: Information on food expenditures, rents, health expenditures, utilities, car-related expenditures, ..... 

• $A_{it}$: Assets holdings are the sum of financial assets, real estate value, pension funds, and car value, net of mortgages and other debt. (Net worth).

• $y_{it}$ are residuals of log total pre-tax household labor earnings on a set of demographics. Note, $c_{it}$ and $a_{it}$ are residuals, using the same set of demographics as for earnings.

▷ cohort and calendar time dummies, family size and composition, education, race, and state dummies.

• As in BPS, we select married male heads aged between 25 and 59.

• In this talk we focus on a balanced sub-sample of $N = 792$ households.
Empirical specification: income

- The quantile function of $\eta_{it}$ given $\eta_{i,t-1}$ is specified as

$$Q_t(\eta_{t-1}, \tau) = Q(\eta_{t-1}, age_t, \tau) = \sum_{k=0}^{K} a_k^Q(\tau) \varphi_k(\eta_{t-1}, age_t),$$

where $\varphi_k$, $k = 0, 1, \ldots, K$, are polynomials (Hermite).

- In addition, the quantile functions of $\varepsilon_{it}$ and $\eta_{i1}$ are

$$Q_{\varepsilon}(age_t, \tau) = \sum_{k=0}^{K} a_k^{\varepsilon}(\tau) \varphi_k(age_t),$$

$$Q_{\eta_1}(age_1, \tau) = \sum_{k=0}^{K} a_k^{\eta_1}(\tau) \varphi_k(age_1).$$
Empirical specification: consumption

• We specify the (log) consumption function as:

\[ g_t(a_t, \eta_t, \varepsilon_t, \tau) = g(a_t, \eta_t, \varepsilon_t, age_t, \tau) \]

\[ = \sum_{k=1}^{K} b_{k}^g \tilde{\phi}_k(a_t, \eta_t, \varepsilon_t, age_t) + b_{0}^g(\tau) \]

– additivity in the taste shifters, though not essential, is convenient given the sample size.

• In addition, the conditional quantiles of \(a_{i1}\) given \(\eta_{i1}\) and \(age_{i1}\) are

\[ Q^{(a)}(\eta_{1}, age_{1}, \tau) = \sum_{k=0}^{K} b_{k}^a(\tau) \tilde{\phi}_k(\eta_{1}, age_{1}) \]
• Model $a^Q_k(\tau)$ as piecewise-linear interpolating splines (Wei and Carroll, 2009) on a grid $0 < \tau_1 < \tau_2 < \ldots < \tau_L < 1$,  
  – convenient as the likelihood function is available in closed form.

• We extend the specification of the intercept coefficient $a_0^Q(\tau)$ on $(0, \tau_1]$ and $[\tau_L, 1)$ using a parametric model: exponential $(\lambda)$.

• In practice, for the PSID data, we take $L = 11$ and $\tau_\ell = \ell / L + 1$. $\varphi_k$ and $\tilde{\varphi}_k$ are low-dimensional tensor products of Hermite polynomials.

• We set $b_0(\tau) = \alpha + \sigma \Phi^{-1}(\tau)$, where $(\alpha, \sigma)$ are to be estimated.
Estimation algorithm

- The first estimation step recovers estimates of the income parameters $\theta$.

- The second step recovers estimates of the consumption parameters $\mu$, given a previous estimate of $\theta$.

- Our choice of a sequential estimation strategy, rather than joint estimation of $(\theta, \mu)$, is motivated by the fact that $\theta$ is identified from the income process alone.
Let $\theta$ be the income-related parameters with true values $\bar{\theta}$.

Let $\rho_\tau(u) = u(\tau - 1\{u \leq 0\})$ denote the “check” function of quantile regression, and let $\overline{a}_{k\ell}^Q$ denote the value of $a_{k\ell}^Q = a_k^Q(\tau_\ell)$ evaluated at the true $\bar{\theta}$. The model implies

$$
\left(\overline{a}_{0\ell}^Q, \ldots, \overline{a}_{K\ell}^Q\right) = \arg\min_{(a_{0\ell}^Q, \ldots, a_{K\ell}^Q)} \mathbb{E} \left[ \int \rho_{\tau_\ell} \left( \eta_{it} - \sum_{k=0}^{K} a_{k\ell}^Q \varphi_k(\eta_{i,t-1}, \text{age}_{it}) \right) f_i(\eta_i^T; \bar{\theta}) d\eta_{iT} \right]
$$

with additional restrictions involving the other parameters in $\theta$.

In the above, $f_i$ denotes the posterior density of $(\eta_{i1}, \ldots, \eta_{iT})$ given the income data

$$
f_i(\eta_i^T; \bar{\theta}) = f(\eta_i^T | y_i^T, \text{age}_i^T; \bar{\theta}).
$$
Model's restrictions: consumption

• Letting $\mu$ (true value $\bar{\mu}$) be the consumption-related parameters, the model implies

$$
\left(\bar{\alpha}, \bar{b}_1^g, \ldots, \bar{b}_K^g\right) = \text{argmin} \quad \mathbb{E} \left[ \int \left( c_{it} - \alpha - \sum_{k=1}^{K} b_k^g \tilde{\varphi}_k(a_{it}, \eta_{it}, y_{it} - \eta_{it}, \text{age}_{it}) \right)^2 g_i(\eta_{iT}; \bar{\theta}, \bar{\mu}) \right],
$$

and

$$
\bar{\sigma}^2 = \mathbb{E} \left[ \int \left( c_{it} - \bar{\alpha} - \sum_{k=1}^{K} b_k^g \tilde{\varphi}_k(a_{it}, \eta_{it}, y_{it} - \eta_{it}, \text{age}_{it}) \right)^2 g_i(\eta_{iT}; \bar{\theta}, \bar{\mu}) d\eta_{iT} \right],
$$

with additional restrictions involving the other parameters in $\mu$.

• Here $g_i$ denotes the posterior density of $(\eta_{i1}, \ldots, \eta_{iT})$ given the earnings, consumption, and asset data

$$
g_i(\eta_{iT}; \bar{\theta}, \bar{\mu}) = f(\eta_{iT} | c_{iT}, a_{iT}, y_{iT}, \text{age}_{iT}; \bar{\theta}, \bar{\mu}).
$$
Overview of estimation

- A compact notation for the restrictions implied by the income model is

\[ \bar{\theta} = \arg\min_{\theta} \mathbb{E} \left[ \int R(y_i, \eta; \theta) f_i(\eta; \bar{\theta}) d\eta \right]. \]

- We use a “stochastic EM” algorithm (in a non-likelihood setup).

Starting with \( \hat{\theta}^{(0)} \) we iterate on \( s=0,1,... \) the following two steps until convergence of the Markov Chain:

1. **Stochastic E-step**: draw \( \eta_i^{(m)} = (\eta_{i1}^{(m)}, ..., \eta_{iT}^{(m)}) \) for \( m = 1, ..., M \) from \( f_i(\cdot; \hat{\theta}^{(s)}) \). ABB use a random-walk Metropolis-Hastings sampler.

2. **M-step**: update

\[ \hat{\theta}^{(s+1)} = \arg\min_{\theta} \sum_{i=1}^{N} \sum_{m=1}^{M} R(y_i, \eta_{i}^{(m)}; \theta). \]
As the likelihood function is available in closed form, the E-step is straightforward.

The M-step consists of a number of ordinary regressions and quantile regressions, such as

$$\min_{(a^Q_0, \ldots, a^Q_K)} \sum_{i=1}^{N} \sum_{t=2}^{T} \sum_{m=1}^{M} \rho_{\tau_\ell} \left( \eta^{(m)}_{it} - \sum_{k=0}^{K} a^Q_{k\ell} \varphi_k(\eta^{(m)}_{i,t-1}, age_{it}) \right), \quad \ell = 1, \ldots, L.$$ 

We compute $\hat{\theta}$ as an average of $\hat{\theta}^{(s)}$ across $S$ iterations.

We estimate $\hat{\theta}$ and $\hat{\mu}$ sequentially.
Statistical properties

• Nielsen (2000) studies the properties of this algorithm in a likelihood case. He provides conditions for the Markov Chain $\hat{\theta}^{(s)}$ to be ergodic (for a fixed sample size).

• He also shows that $\sqrt{N} \left( \hat{\theta}^{(s)} - \bar{\theta} \right)$ converges to a Gaussian autoregressive process as $N$ tends to infinity.

• Arellano and Bonhomme [AB] (2015) adapt Nielsen’s arguments to derive the form of the asymptotic variance in a non-likelihood case.

• AB also study consistency as $K$ (number of polynomial terms) and $L$ (number of knots) tend to infinity with $N$. 
Empirical results
Nonlinear persistence of $\eta_{it}$ (PSID):

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_{\eta_t|\eta_{t-1}}(\eta_{i,t-1}, \tau)}{\partial \eta}$$

Note: Estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $\eta_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the distribution of $\eta_{i,t-1}$. Evaluated at mean age in the sample.
Nonlinear persistence of $y_{it}$

$$\frac{\partial Q_{yt|y_{t-1}}(y_{i,t-1},\tau)}{\partial y}$$

PSID panel data

Nonlinear model

Note: Estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $y_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the dist. of $y_{i,t-1}$. 
Nonlinear persistence of $y_{it}$

$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1}, \tau)}{\partial y}$$

Norwegian register data

Nonlinear model

Note: Estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $y_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the dist. of $y_{i,t-1}$. 
Figure: Densities of persistent and transitory earnings components (PSID)

(a) Persistent component $\eta_{it}$  
(b) Transitory component $\varepsilon_{it}$

Note: Nonparametric estimates of densities based on simulated data according to the nonlinear model, using a Gaussian kernel.
Nonlinear persistence of $y_{it}$ (cont.)

$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1}, \tau)}{\partial y}$$

**Note:** Estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $y_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the dist. of $y_{i,t-1}$. 

PSID data

Canoncal model
Nonlinear persistence, 95% confidence bands

(a) Earnings, PSID data  
(b) Earnings, nonlinear model

Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.
Figure: Conditional skewness of log-earnings residuals and $\eta$ component

(a) Log-earnings residuals $y_{it}$

(b) Persistent component $\eta_{it}$

Note: Conditional skewness $sk(y, \tau)$ and $sk(\eta, \tau)$, for $\tau = 11/12$. Log-earnings residuals (left) and $\eta$ component (right). The $x$-axis shows the conditioning variable, the $y$-axis shows the corresponding value of the conditional skewness measure. Bootstrap confidence intervals in the Appendix.
Consumption response to $\eta_{it}$, by assets and age

$$\bar{\phi}_t(a) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta_{it}, \epsilon_{it}, \nu_{it})}{\partial \eta_{it}} \right], \text{ nonlinear model}$$

Note: Estimates of the average consumption response $\bar{\phi}_t(a)$ to variations in $\eta_{it}$, evaluated at $\tau_{\text{assets}}$ and $\tau_{\text{age}}$. 
Consumption responses to $y_{it}$, by assets and age

$$\mathbb{E} \left[ \frac{\partial}{\partial y} \bigg|_{y_{it}} \mathbb{E} \left( c_{it} \big| a_{it} = a, y_{it} = y, age_{it} = age \right) \right]$$

**PSID data**

**Nonlinear model**

Note: Estimates of the average derivative of the conditional mean of $c_{it}$ given $y_{it}$, $a_{it}$ & $age_{it}$ with respect to $y_{it}$, evaluated at values of $a_{it}$ & $age_{it}$ corresponding to their $\tau_{assets}$ & $\tau_{age}$ percentiles, and averaged over the values of $y_{it}$. 
Figure: Household heterogeneity in earnings

(a) Nonlinear persistence of $\eta_{it}$  

(b) Conditional skewness of $\eta_{it}$

Notes: (a) Estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, based on estimates from the nonlinear earnings model with an additive household-specific effect.

(b) Conditional skewness $sk(\eta, \tau)$, for $\tau = 11/12$, based on the same model.
Consumption response to $\eta_{it}$, by assets and age, household heterogeneity

$$\overline{\phi}_t(a) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta_{it}, \xi_{it}, \nu_{it})}{\partial \eta} \right], \text{ nonlinear model}$$

Note: Estimates of the average consumption response $\overline{\phi}_t(a)$ to variations in $\eta_{it}$, evaluated at $\tau_{assets}$ and $\tau_{age}$. 
Model’s simulation

• Simulate life-cycle earnings and consumption after a shock to the persistent earnings component (at age 37).

• We report the difference between:
  – Households that are hit by a “bad” shock ($\tau_{\text{shock}} = .10$) or by a “good” shock ($\tau_{\text{shock}} = .90$).
  – Households that are hit by a median shock $\tau = .5$.

• Age-specific averages across 100,000 simulations. At age 35 all households have the same persistent component (percentile $\tau_{\text{init}}$).
Impulse responses, earnings

Bad shock: $\tau_{\text{shock}} = .1$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$

Good shock: $\tau_{\text{shock}} = .9$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$
Impulse responses, consumption

Bad shock: $\tau_{\text{shock}} = .1$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$

Good shock: $\tau_{\text{shock}} = .9$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$
Impulse responses, consumption, household heterogeneity

**Bad shock:** $\tau_{\text{shock}} = .1$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$

**Good shock:** $\tau_{\text{shock}} = .9$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$
Impulse responses, consumption, linear assets rule

Nonlinear model $\tau_{init} = .1$

(a) $\tau_{shock} = .1$

(b) $\tau_{shock} = .9$

$\tau_{init} = .9$

(e) $\tau_{shock} = .1$

(f) $\tau_{shock} = .9$

Note: Linear assets accumulation rule. Assets are constrained to be non-negative.
Impulse responses: canonical earnings and linear consumption model

Earnings

\[ \tau_{\text{shock}} = .1 \]

\[ \tau_{\text{shock}} = .9 \]

Consumption

\[ \tau_{\text{shock}} = .1 \]

\[ \tau_{\text{shock}} = .9 \]
Impulse responses, by age and initial assets

**Earnings**

\[ \tau_{init} = .9, \quad \tau_{shock} = .1 \]

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- Log-earnings vs. age

\[ \tau_{init} = .1, \quad \tau_{shock} = .9 \]

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- Log-earnings vs. age

**Consumption**

\[ \tau_{init} = .9, \quad \tau_{shock} = .1 \]

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- Log-consumption vs. age

\[ \tau_{init} = .1, \quad \tau_{shock} = .9 \]

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- Log-consumption vs. age

Notes: Initial assets at age 35 (for “young” households) or 51 (for “old” households) are at percentile .10 (dashed curves) and .90 (solid curves). Linear assets accumulation rule. Assets are constrained to be non-negative.
Summary

• New framework to shed new light on the nonlinear transmission of income shocks to consumption and the nature of insurance to income shocks.
Summary

- New framework to shed new light on the nonlinear transmission of income shocks to consumption and the nature of insurance to income shocks.
  - A Markovian permanent-transitory model of household income, which reveals asymmetric persistence of unusual shocks in the PSID and in large administrative registers.
  - An age-dependent nonlinear consumption rule that is a function of assets, permanent income and transitory income.
Summary

- New framework to shed new light on the nonlinear transmission of income shocks to consumption and the nature of insurance to income shocks.

- A Markovian permanent-transitory model of household income, which reveals asymmetric persistence of unusual shocks in the PSID and in large administrative registers.

- An age-dependent nonlinear consumption rule that is a function of assets, permanent income and transitory income.

- Provide conditions for nonparametric identification:
  - Explain how a simulation-based sequential QR method is feasible.

- This framework leads to new empirical measures of the degree of partial insurance and the link between income and consumption inequality.

- But what about looking inside the family labour income measure ......?
Families have the possibility of insuring consumption on many margins.

Distinguish four separate mechanisms:
A role for family labour supply?

Families have the possibility of insuring consumption on many margins.

**Distinguish four separate mechanisms:**

1. Labor supply of other family members,
2. Non-linear taxes and welfare,
3. **Self-insurance** (i.e., savings through the direct use of net assets),
4. Other informal mechanisms and networks....

- Then examine each step in the distributional dynamics from wages to consumption:

  \[
  \text{wages} \rightarrow \text{earnings} \rightarrow \text{family earnings} \rightarrow \text{net income} \rightarrow \text{consumption} \rightarrow \text{wealth}.
  \]
A role for family labour supply?

BPS use data on wage, consumption, income, labor supply and assets from the PSID.

As described in the *Nemmers Lecture*, BPS show that family labor supply can be a key mechanism for ‘insuring’ unexpected shocks

- especially for younger families and for those with limited access to assets,
- a strong “added-worker” effect as a response to a permanent shock.

* Find an important role for unusual shocks and nonlinear persistence in the wages......
Recent research (BPS2) combines data on wage, consumption, income, labor supply, assets and *time-use* from the PSID, ATUS and CEX.

- Time-use data from ATUS is used to unpack what’s going on in terms of family time allocation responses to male and female wage shocks.

  - results uncover a tension between the desire of spouses to spend leisure time with each other, and the specialization in care of children.

  - the presence of young children is found to give rise to Frisch substitutability of hours between spouses, with a switch to Frisch complements as children age and leave home.
Recent research (BPS2) combines data on wage, consumption, income, labor supply, assets and time-use from the PSID, ATUS and CEX.

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  - results uncover a tension between the desire of spouses to spend leisure time with each other, and the specialization in care of children.

  - the presence of young children is found to give rise to Frisch substitutability of hours between spouses, with a switch to Frisch complements as children age and leave home.

The strong “added-worker” effect from a response to an adverse permanent shock to his earnings is found to induce a fall in mother’s time-use with young children, especially for low-educated with low assets.

- Details of the family labour supply, time-use and consumption smoothing model and results at the end of these lecture slides.
Next steps

1. Study the implications for child outcomes, currently linking to \textit{CDS}.
2. Separate housing equity and allow a role for local labour markets.
3. Include firm to firm transitions and lay-offs.
4. Include experience/human capital $\Rightarrow$ as in BDMS (\textit{Ecta} 2016).
5. Health and other types of (partially insured) shocks (\textit{HRS, ELSA}).
6. Estimate on the full population (Norwegian) register data.
7. and more......
• We work in $L^2$-spaces relative to suitable distributions.
• Let $g(y_2, y_3)$ such that there exists a $s(y_2)$ such that

$$
\mathbb{E}[g(Y_2, Y_3) | Y_1] = \mathbb{E}[s(Y_2) | Y_1].
$$

Under completeness of $Y_2 | Y_1$, $s(\cdot)$ is unique.
• By conditional independence,

$$
\mathbb{E}[\mathbb{E}(g(Y_2, Y_3) | \eta_2) | Y_1].
$$

• Under completeness of $\eta_2 | Y_1$, it follows that

$$
\mathbb{E}[g(Y_2, Y_3) | \eta_2] = \mathbb{E}[s(Y_2) | \eta_2].
$$
The case $T = 3$ (cont.)

- Wilhelm (12) considers the functions $g_1(Y_3) = 1\{Y_3 \leq y_3\}$, and $g_2(Y_2, Y_3) = Y_2 1\{Y_3 \leq y_3\}$, for a given value $y_3$.
- This yields

\[ \mathbb{E} [1\{Y_3 \leq y_3\} | \eta_2] = G(\eta_2) = \mathbb{E} [s_1(Y_2) | \eta_2] \]

\[ \mathbb{E} [Y_2 1\{Y_3 \leq y_3\} | \eta_2] = \eta_2 G(\eta_2) = \mathbb{E} [s_2(Y_2) | \eta_2]. \]

- Hence, taking Fourier transforms (i.e., $\mathcal{F}(h)(u) = \int h(x) e^{iux} dx$),

\[ \mathcal{F}(G)(u) = \mathcal{F}(s_1)(u) \psi_{\varepsilon_2}(-u) \]

\[ i^{-1} d\mathcal{F}(G)(u) / du = \mathcal{F}(s_2)(u) \psi_{\varepsilon_2}(-u), \]

where $\psi_{\varepsilon_2}(u) = \mathcal{F}(f_{\varepsilon_2})(u)$ is the characteristic function of $\varepsilon_2$, and $i = \sqrt{-1}$. 
The case $T = 3$ (cont.)

This yields the following first-order differential equation

$$F_2(u) du = \left[ \frac{dF(s_1)(-u)}{du} - iF(s_2)(-u) \right] \psi_{\varepsilon_2}(u).$$

- In addition, $\psi_{\varepsilon_2}(0) = 1$.
- This ODE can be solved in closed form for $\psi_{\varepsilon_2}(\cdot)$, provided that $F(s_1)(u) \neq 0$ for all $u$ (which is another injectivity condition).
- As a result, the distribution of $\varepsilon_2$, and the distribution of $Y_3$ given $\eta_2$, are both nonparametrically identified.
### Descriptive statistics PSID (means)

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<td>Consumption</td>
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<td>376,485</td>
<td>399,901</td>
<td>501,590</td>
<td>460,262</td>
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</table>

**Notes: Balanced subsample from PSID, N = 749, T = 6.**

- Compared to BPS (12), households in our balanced sample have higher assets, and to a less extent higher earnings and consumption.
Consumption response, two-period model

- CRRA utility. The Euler equation is (assuming $\beta(1 + r) = 1$)

\[ C_1^{-\gamma} = \mathbb{E}_1 \left[ ((1 + r)A_2 + Y_2)^{-\gamma} \right], \]

where $\gamma > 0$ is risk aversion and we have used the budget constraint $A_3 = (1 + r)A_2 + Y_2 - C_2 = 0$.

- Let $X_1 = (1 + r)A_1 + Y_1$, $R = (1 + r)X_1 + \mathbb{E}_1(Y_2)$, and $Y_2 = \mathbb{E}_1(Y_2) + \sigma W$. Expanding as $\sigma \to 0$ we obtain

\[ C_1 \approx \frac{(1 + r)X_1 + \mathbb{E}_1(Y_2)}{2 + r} - \frac{\gamma + 1}{2R} \mathbb{E}_1(W^2) + \frac{(2 + r)(\gamma + 1)(\gamma + 2)}{6R^2} \mathbb{E}_1(W^3). \]

- certainty equivalent
- precautionary-variance
- precautionary-skewness
Nonlinear persistence, 95% confidence bands

(a) Earnings, PSID data  (b) Earnings, nonlinear model

(c) Persistent component $\eta_{it}$, nonlinear model

Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.
Conditional skewness of log-earnings residuals and $\eta$ component, 95% confidence bands

(a) Log-earnings residuals $y_{it}$  (b) Persistent component $\eta_{it}$

Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.
Nonlinear persistence of $\eta_{it}$ (Norwegian register data):

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_{\eta_t|\eta_{t-1}}(\eta_{i,t-1}, \tau)}{\partial \eta}$$

Note: Estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $\eta_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the distribution of $\eta_{i,t-1}$. Evaluated at mean age in the sample.
Family Labour Supply, Time-Use and Consumption Smoothing Modelling Slides
**Some Related Literature**


BPS Model
Model Setup and Solution

A life-cycle model of family labour supply, time-use and consumption decisions with:

- Two earners using their time for leisure/input for child production function/work.
- Wage uncertainty for two earners (transitory and persistent).
MODEL SETUP AND SOLUTION

A life-cycle model of family labour supply, time-use and consumption decisions with:

- Two earners using their time for leisure/input for child production function/work.
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From model to estimation:
Mix semi-structural methods with structural dynamic programming:

1. Semi-structural estimation of a subset of utility and production parameters. Use MRS to derive analytical estimation equations $\implies$ estimate a subset of parameters using PSID, CEX and ATUS.
MODEL SETUP AND SOLUTION

A life-cycle model of family labour supply, time-use and consumption decisions with:

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From model to estimation:
Mix semi-structural methods with structural dynamic programming:

1. Semi-structural estimation of a subset of utility and production parameters. Use MRS to derive analytical estimation equations \(\rightarrow\) estimate a subset of parameters using PSID, CEX and ATUS.

2. Structural dynamic model to capture life-cycle dynamics, uncertainty and borrowing constraints. Solve the model numerically given the parameters from stage 1. \(\rightarrow\) estimate the remaining parameters using SMM, and provide counterfactual simulations (persistent shock etc.)
Household Lifecycle (Baseline) Model

Household chooses \( \{C_{t+s}, L_{1,t+s}, L_{2,t+s}, T_{1,t+s}, T_{2,t+s}\} \) to maximize:

\[
\mathbb{E}_t \sum_{s=0}^{T-t} u_{t+s} (C_{t+s}, L_{1,t+s}, L_{2,t+s}, T_{1,t+s}, T_{2,t+s}; z_{t+s}, \epsilon)
\]

subject to

\[
A_{t+1} = (1 + r) (A_t + \mathcal{T}(z_{t}, H_{1,t} W_{i,1,t} + H_{2,t} W_{i,2,t}) - C_t)
\]

\[
L_{1,t} + H_{1,t} + T_{1,t} = \bar{L}, \quad H_{2,t} + L_{2,t} + T_{2,t} = \bar{L}
\]

\[
H_{j,t} \geq 0, L_{j,t} \geq 0, T_{j,t} \geq 0, \quad A_{t+1} \geq 0, \quad A_{T+1} = 0
\]

C: consumption

\(L_j\): leisure time of earner \(j\), \(T_j\): parental time of earner \(j\)

\(\bar{L}\): maximum time available for work, leisure, childcare

\(z\): demographic characteristics, \(\epsilon\): unobserved heterogeneity

\(A_t\): assets at the beginning of the period

\(r\): nonstochastic interest rate

\(W_{j,t}\): hourly wages

\(\mathcal{T}(.)\): nonlinear tax function.
Uncertainty in Wages

- Assume the log of real wage of earner $j = \{1, 2\}$ at age $t$ can be written as:

\[
\log W_{j,t} = x'_{j,t} \beta^j_W + \eta_{j,t} + u_{j,t} 
\]

\[
\eta_{j,t} = \eta_{j,t-1} + v_{j,t} 
\]

- Shocks can be correlated across spouses
- $x'_{i,j,t}$: Observed characteristics (e.g. age, state of residence etc.). Assumed to be known to the household.
Identification (Wage Parameters)

From:

\[ \Delta w_{i,j,t} = \Delta u_{i,j,t} + v_{i,j,t} \]

It follows that:

\[
\begin{align*}
\sigma_{u_j}^2 &= -E(\Delta w_{i,j,t}\Delta w_{i,j,t+1}) \\
\sigma_{v_j}^2 &= E(\Delta w_{i,j,t}(\Delta w_{i,j,t+1} + \Delta w_{i,j,t} + \Delta w_{i,j,t-1})) \\
\sigma_{u_i u_{-j}} &= -E(\Delta w_{i,j,t}\Delta w_{i,-j,t+1}) \\
\sigma_{v_i v_{-j}} &= E(\Delta w_{i,j,t}(\Delta w_{i,-j,t+1} + \Delta w_{i,-j,t} + \Delta w_{i,-j,t-1}))
\end{align*}
\]
Identification (Wage Parameters)

From:

\[ \Delta w_{i,j,t} = \Delta u_{i,j,t} + v_{i,j,t} \]

It follows that:

\[ \sigma^2_{u_j} = -E(\Delta w_{i,j,t}\Delta w_{i,j,t+1}) \]
\[ \sigma^2_{v_j} = E(\Delta w_{i,j,t}(\Delta w_{i,j,t+1} + \Delta w_{i,j,t} + \Delta w_{i,j,t-1})) \]
\[ \sigma_{u_ju_{-j}} = -E(\Delta w_{i,j,t}\Delta w_{i,-j,t+1}) \]
\[ \sigma_{v_jv_{-j}} = E(\Delta w_{i,j,t}(\Delta w_{i,-j,t+1} + \Delta w_{i,-j,t} + \Delta w_{i,-j,t-1})) \]

- Easily adapted for dynamic quantile model with nonlinear persistence is particularly well suited to our mixed quasi-structural/dynamic programming approach.
Two earner household utility within period $t$ (baseline spec):

$$u(.) = \phi_{C,i} \frac{\tilde{C}_i^{1-1/\eta_{c,p}}}{1 - 1/\eta_{c,p}} - \frac{1}{1 - \rho_L} \left( \phi_{L_1,i} L_{1,i}^{1-1/\varphi_{L_1}} + \phi_{L_2,i} L_{2,i}^{1-1/\varphi_{L_2}} \right)^{1-\rho_L}$$

$$- \frac{1}{1 - \rho_T} \left( \phi_{T_1,i} T_{1,i}^{1-1/\varphi_{T_1}} + \phi_{T_2,i} T_{2,i}^{1-1/\varphi_{T_2}} \right)^{1-\rho_T}$$

$$0 < \phi_{0,i}, \phi_{L_1,i}, \phi_{L_2,i}, \phi_{T_1,i}, \phi_{T_2,i}, 0 < \varphi_{L_1}, \varphi_{L_2} < 1, \rho_L, \rho_T < 1$$

Allow marginal utility of consumption to shift with employment (nonseparability), let $\tilde{C} = e^{\gamma E_2} C_{t+s}$ where $E_2 = 1\{H_2 > 0\}$. 
Two earner household utility within period $t$ (baseline spec):

$$u(.) = \phi_{C,i} \frac{\tilde{C}^{1-1/\eta_{c,p}}_{i,t}}{1-1/\eta_{c,p}} - \frac{1}{1-\rho_L} \left( \phi_{L_1,i} L_{1,i}^{1-1/\varphi_{L_1}} + \phi_{L_2,i} L_{2,i}^{1-1/\varphi_{L_2}} \right)^{1-\rho_L} - \frac{1}{1-\rho_T} \left( \phi_{T_1,i} T_{1,i}^{1-1/\varphi_{T_1}} + \phi_{T_2,i} T_{2,i}^{1-1/\varphi_{T_2}} \right)^{1-\rho_T} \quad 0 < \varphi_{L_1}, \varphi_{L_2} < 1, \rho_L, \rho_T < 1$$

Allow marginal utility of consumption to shift with employment (nonseparability), let $\tilde{C} = e^{\gamma E_2 C_{t+s}}$ where $E_2 = 1\{H_2 > 0\}$.

Utility shifters for good $x$: $\phi_{x,i} = f_x (z_{i,t}, \varepsilon_{x,i,t}, \zeta_{x,i})$

$z$: includes children characteristics,
$\varepsilon, \zeta_{x,i}$: unobserved stochastic heterogeneity components.
For interior solutions:

\[ L_2 = \left( \frac{W_{1,t}}{{W_{2,t}}} \frac{\tilde{\phi}_L}{\hat{\phi}_L} L_1^{1/\varphi_L} \right)^{\varphi_{L2}} \]

\[ L_2 = \left[ W_{2,t} \frac{\phi_C}{\tilde{\phi}_L} C^{-1/\eta_{c,p}} \left( \frac{1}{\varphi_L} \left( \frac{\phi_L L_1}{L_2} \right)^{1-1/\varphi_L} \right)^{\rho_L} \right]^{-\varphi_{L2}} \]

where \( \tilde{\phi}_x \equiv \phi_x (1/\varphi_x - 1) \)

Note: similar relation for parental time use \( T_1, T_2 \).

\( \rho_L > 0 \) implies Frisch complement.

Assume preference heterogeneity and shifts in marginal utility can be written as:

\[ \log \left( \tilde{\phi}_{x,i,t} \right) = \bar{\tilde{\phi}}_x (z_{i,t}) + \varepsilon_{x,i,t} + \zeta_{x,i} \]
Imply log-linear quasi-structural estimation equations:

\[ l_2 = K_0 + \varphi_{L_2} (w_1 - w_2) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \nu_1 \]

\[ l_2 = K_1 - \varphi_{L_2} w_2 + \frac{\varphi_{L_2}}{\eta_{c,p}} c_t + \frac{\varphi_{L_2}}{\varphi_{L_1}} \rho_L \left(1 - \varphi_{L_1}\right) l_1 - \varphi_{L_2} \rho_L M + \nu_3 \]

where: \[ M = \frac{\varphi_{L_2}}{\varphi_{L_1}} \frac{1 - \varphi_{L_1}}{1 - \varphi_{L_2}} \frac{W_2 L_2}{W_1 L_1} \]

(lower case for log(), and omitting time and household subscripts).

- \( K' \)’s are deterministic, and \( \nu' \)’s are linear combinations of \( \varepsilon' \)’s and \( \zeta' \)’s.
- Analogous equations for child time use inputs \( t_2 \).
Using MRS to Recover Preference Parameters

- Imply log-linear quasi-structural estimation equations:

\[
\begin{align*}
    l_2 &= K_0 + \varphi_{L_2} (w_1 - w_2) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \nu_1 \\
    l_2 &= K_1 - \varphi_{L_2} w_2 + \frac{\varphi_{L_2}}{\eta_{c,p}} c_t + \frac{\varphi_{L_2}}{\varphi_{L_1}} \rho_{L} \left(1 - \varphi_{L_1}\right) l_1 - \varphi_{L_2} \rho_{L} M + \nu_3
\end{align*}
\]

where: 

\[
M = \frac{\varphi_{L_2} \left(1 - \varphi_{L_1}\right)}{\varphi_{L_1} \left(1 - \varphi_{L_2}\right)} \frac{W_2 L_2}{W_1 L_1}
\]

(lower case for log(), and omitting time and household subscripts).

- \(K\)'s are deterministic, and \(\nu\)'s are linear combinations of \(\varepsilon\)'s and \(\zeta\)'s.
- Analogous equations for child time use inputs \(t_2\).
- Imply nonlinear panel data moment conditions (with appropriate instruments, and participation condition), to consistently estimate \(\varphi_{L_1}, \varphi_{L_2}, \rho_{L}, \eta_{c,p}, \varphi_{T_1}, \varphi_{T_2}, \) and \(\rho_{T}\) by nonlinear GMM.
• Imply log-linear quasi-structural estimation equations:

\[
\begin{align*}
    l_2 &= K_0 + \varphi_{L2} (w_1 - w_2) + \frac{\varphi_{L2}}{\varphi_{L1}} l_1 + \nu_1 \\
    l_2 &= K_1 - \varphi_{L2} w_2 + \frac{\varphi_{L2}}{\eta_{c,p}} c_t + \frac{\varphi_{L2}}{\varphi_{L1}} \rho_L \left(1 - \varphi_{L1}\right) l_1 - \varphi_{L2} \rho_L M + \nu_3
\end{align*}
\]

where: \(M = \frac{\varphi_{L2} \left(1 - \varphi_{L1}\right)}{\varphi_{L1} \left(1 - \varphi_{L2}\right)} \frac{W_2 L_2}{W_1 L_1}\)

(lower case for log(), and omitting time and household subscripts).

• \(K\)'s are deterministic, and \(\nu\)'s are linear combinations of \(\varepsilon\)'s and \(\zeta\)'s.

• Analogous equations for child time use inputs \(t_2\).

• Imply nonlinear panel data moment conditions (with appropriate instruments, and participation condition), to consistently estimate \(\varphi_{L1}, \varphi_{L2}, \rho_L, \eta_{c,p}, \varphi_{T1}, \varphi_{T2}\), and \(\rho_T\) by nonlinear GMM.

• But need to recover, fixed cost parameter(s) \(\gamma, \tilde{\phi}_x (z_{i,t})\) and the distribution of unobserved heterogeneity and taste shocks.
Structural model is also required for counterfactual simulations.

Method:

- Solve the stochastic life cycle problem given \( \varphi_{L1}, \varphi_{L2}, \rho_L, \eta_{c,p}, \varphi_{T1}, \varphi_{T2}, \) and \( \rho_T \), and use SMM to complete the estimation.

Moments to target include:

- Distribution of hours and time spent with children of each earner at different points over the life-cycle.
- Levels of employment and employment/non-employment transitions.
- Consumption changes with children.

How does this ‘mixed’ structural approach this compare with the ‘partial insurance’ approximations?
Comparison with Partial Insurance Approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

\[
\begin{pmatrix}
\Delta h_{1,t} \\
\Delta h_{2,t} \\
\Delta c_t
\end{pmatrix} \simeq \Theta X +
\begin{pmatrix}
\kappa_{h1,u1} & 0 & \kappa_{h1,v1} & \kappa_{h1,v2} \\
0 & \kappa_{h2,u2} & \kappa_{h2,v1} & \kappa_{h2,v2} \\
0 & 0 & \kappa_{c,v1} & \kappa_{c,v2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
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\end{pmatrix}
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\[\kappa_{h_j,u_j} = \eta_{h_j,w_j} \rightarrow [\text{Frisch}]\]
Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

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\Delta u_{2,t} \\
v_{1,t} \\
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\[\kappa_{h_j,u_j} = \eta_{h_j,w_j} \rightarrow \text{[Frisch]} \quad \kappa_{h_j,v_j} \rightarrow \text{[Marshall]}\]
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\]

\[
\kappa_{c,v_j} = (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}}
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\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

\[
\kappa_{h_j,u_j} = \eta_{h_j,w_j} \quad \text{[Frisch]} \quad \kappa_{h_j,v_j} \quad \text{[Marshall]} \quad \kappa_{h_j,v_{-j}} \quad \text{[AWE]}
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\[
\kappa_{c,v_j} = (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}}
\]

\[
\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}
\]
Comparison with Partial Insurance Approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

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\end{pmatrix}
\begin{pmatrix}
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\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

\[
\kappa_{h_j,u_j} = \eta_{h_j,w_j} \rightarrow \text{[Frisch]} \hspace{1cm} \kappa_{h_j,v_j} \rightarrow \text{[Marshall]} \hspace{1cm} \kappa_{h_j,v_{-j}} \rightarrow \text{[AWE]}
\]

\[
\kappa_{c,v_j} = (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}}
\]

\[
s_{i,j,t} \approx \frac{\text{Human Wealth}_{i,j,t}}{\text{Human Wealth}_{i,t}}
\]
Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

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- \( \kappa_{c,v_j} = (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} (1 + \eta_{h_j,w_j})}{\eta_{c,p} + \left(1 - \pi_{i,t}\right) \eta_{h,w}} \)

- \( \eta_{c,p} \rightarrow \text{Consumption EIS} \)
Comparison with partial insurance approach

- Use approximation of FOCs (under separability) and of lifetime budget constraint

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\[\kappa_{c,v_j} = (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}}\]

\[\overline{\eta}_{h,w} = s_{i,j,t} \eta_{h_j,w_j} + s_{i,-j,t} \eta_{h_{-j},w_{-j}}\]
**Comparison with Partial Insurance Approach**

- Use approximation of FOCs (under separability) and of lifetime budget constraint

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\begin{pmatrix}
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\]

\[
\kappa_{c,v_j} = (1 - \beta) (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + (1 - \beta) (1 - \pi_{i,t}) \eta_{h,w}}
\]
**Comparison with Partial Insurance Approach**

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v_{1,t} \\
v_{2,t}
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\]

\[
\kappa_{h_j,u_j} = \eta_{h_j,0} \rightarrow \text{[Frisch]} \quad \kappa_{h_j,v_j} \rightarrow \text{[Marshall]} \quad \kappa_{h_j,v_{-j}} \rightarrow \text{[AWE]}
\]

\[
\kappa_{c,v_j} = (1 - \beta) (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,0}\right)}{\eta_{c,p} + (1 - \beta) (1 - \pi_{i,t}) \eta_{h,w}}
\]

\[
\beta \rightarrow \text{External insurance (networks, etc.)}
\]
The share of assets to human wealth by age

\[ \pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}} \]

Source: Blundell, Pistaferri and Saporta-Eksten (2016)
The distribution of shares of assets to human wealth by age

\[ \pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}} \]

Source: Blundell, Pistaferri and Saporta-Eksten (2016)
When preferences are non-separable:

\[
\begin{pmatrix}
\Delta h_{1,t} \\
\Delta h_{2,t} \\
\Delta c_t
\end{pmatrix} \sim \Theta X +
\begin{pmatrix}
\kappa_{h_1,u_1} & \kappa_{h_1,u_1} & \kappa_{h_1,v_1} & \kappa_{h_1,v_1} \\
\kappa_{h_2,u_1} & \kappa_{h_2,u_2} & \kappa_{h_2,v_1} & \kappa_{h_2,v_2} \\
\kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

- \( \kappa_{c,u_j} \rightarrow \) non-separability between consumption and leisure of member \( j \)
  - Identified by response of consumption to transitory shock having no wealth effects
- \( \kappa_{h_{j,u_{-j}}} \rightarrow \) non-separability between spouses’ leisures
  - Identified by response of member \( j \)’s labor supply to transitory shock faced by spouse

BPS estimates suggest Frisch substitutes for families with younger children.

But, as with other similar semi-structural methods, insufficient to identify intertemporal/life-cycle counterfactuals.
\[ \begin{pmatrix} \Delta c_{\tau} \\
\Delta l_{1,\tau} \\
\Delta l_{2,\tau} \\
\Delta t_{1,\tau} \\
\Delta t_{2,\tau} \end{pmatrix} \sim \begin{pmatrix} \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{t_1^l,u_1} & \kappa_{t_1^l,u_2} & \kappa_{t_1^l,v_1} & \kappa_{t_1^l,v_2} \\
\kappa_{t_2^l,u_1} & \kappa_{t_2^l,u_2} & \kappa_{t_2^l,v_1} & \kappa_{t_2^l,v_2} \\
\kappa_{t_1^c,u_1} & \kappa_{t_1^c,u_2} & \kappa_{t_1^c,v_1} & \kappa_{t_1^c,v_2} \\
\kappa_{t_2^c,u_1} & \kappa_{t_2^c,u_2} & \kappa_{t_2^c,v_1} & \kappa_{t_2^c,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t} \end{pmatrix} \]

where \( \kappa_{m,v_j} = \kappa_{m,v_j}^{m,v_j}(\pi_t, s_t, \eta, T()) \),

and \( s_t \approx \frac{\text{Human Wealth}_{1,t}}{\text{Human Wealth}_t} \), \( \pi_t \approx \frac{\text{Assets}_t}{\text{Assets}_t + \text{Human Wealth}_t} \).

This quasi-structural approach performs less well near/at corners.

It cannot recover fixed cost/extensive margin parameters and cannot simulate counterfactuals.

However, useful ‘moments’ to simulate in the structural model to examine ‘Frisch’ and ‘Marshallian’ responses.
Data and Estimation
Estimating the MRS equations requires data on:

- Leisure, parental time and hourly wages of both earners.
- Household consumption and assets.
- Family composition.
- Valid instruments for endogenous variables (consumption, leisure etc.), and wages (due to measurement error and selection).
Estimating the MRS equations requires data on:

- Leisure, parental time and hourly wages of both earners.
- Household consumption and assets.
- Family composition.
- Valid instruments for endogenous variables (consumption, leisure etc.), and wages (due to measurement error and selection).

Where can we find such data?...

- PSID:
  - Unique panel data on consumption, assets, hours of work and hourly wages of both earners (biennial since 1999).
  - Very noisy parental time use measures in CDS diary (used in previous work for this paper).

- ATUS: Detailed time use data; (annual since 2003).
- CEX: Detailed consumption data (to match annual ATUS).
Combine the Multiple Sources

1. Use PSID (1999-2009/2013) to estimate the MRS equations for leisure by GMM for families with no young children
   \( \Rightarrow \varphi_{L1}, \varphi_{L2}, \rho_L, \eta_{cp} \).

2. For the moment estimator of the MRS equations for parental time:
   - Use ATUS (2003-2014) parental time of married women with young children combined with hourly wages of both spouses. Combine with cohort-education-year aggregate of husband parental time (ATUS), and consumption (CEX).
     \( \Rightarrow \varphi_{T1}, \varphi_{T2}, \rho_T \).
   - Note: apply similar sample selection in all datasets:
     - Married couples, wife aged 25-64.
     - In GMM, condition on employment of both earners (and apply correction).
### Descriptives of Time Use data in the ATUS

<table>
<thead>
<tr>
<th>Description</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-zero childcare time (head)</td>
<td>0.69</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-zero childcare time (wife)</td>
<td>0.91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Childcare annual hours (head) inc. 0s</td>
<td>320</td>
<td>0</td>
<td>195</td>
<td>498</td>
</tr>
<tr>
<td>Childcare annual hours (wife) inc. 0s</td>
<td>709</td>
<td>260</td>
<td>585</td>
<td>1,023</td>
</tr>
<tr>
<td>Childcare annual hours (head) exc. 0s</td>
<td>466</td>
<td>182</td>
<td>355</td>
<td>628</td>
</tr>
<tr>
<td>Childcare annual hours (wife) exc. 0s</td>
<td>778</td>
<td>347</td>
<td>650</td>
<td>1,070</td>
</tr>
</tbody>
</table>

Notes: ATUS data from 2003-2014 for the sample of married couples, wife aged 25-65 with youngest child aged 10 or less.
## Descriptives of Consumption, Leisure and Wages in the PSID

<table>
<thead>
<tr>
<th></th>
<th>(1) mean</th>
<th>(2) p25</th>
<th>(3) median</th>
<th>(4) p75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Consumption (exc. durables)</td>
<td>40,997</td>
<td>26,237</td>
<td>35,654</td>
<td>49,307</td>
</tr>
<tr>
<td>Hours of husband</td>
<td>2,011</td>
<td>1,835</td>
<td>2,080</td>
<td>2,500</td>
</tr>
<tr>
<td>Hours of wife</td>
<td>1,349</td>
<td>347</td>
<td>1,645</td>
<td>2,016</td>
</tr>
<tr>
<td>Hourly wage of husband</td>
<td>31.3</td>
<td>15.2</td>
<td>22.6</td>
<td>34.8</td>
</tr>
<tr>
<td>Hourly wage of wife</td>
<td>21.3</td>
<td>11.4</td>
<td>17.3</td>
<td>26.3</td>
</tr>
</tbody>
</table>

Notes: PSID data from 1999-2013 PSID waves, for the sample of married couples, wife aged 25-65 with youngest child aged 10 or less. Consumption and wages in 2010 prices. In computations leisure time is calculated assuming total hours is 4160 (5*16*52).
Results
### MRS Estimates

<table>
<thead>
<tr>
<th></th>
<th>(1) PSID</th>
<th>(2) ATUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_{L_1}$</td>
<td>0.161***</td>
<td>$\varphi_{T_1}$</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td></td>
</tr>
<tr>
<td>$\varphi_{L_2}$</td>
<td>0.115***</td>
<td>$\varphi_{T_2}$</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td></td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>0.646***</td>
<td>$\rho_T$</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td></td>
</tr>
<tr>
<td>$\eta_{cp}$</td>
<td>0.807***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td></td>
</tr>
<tr>
<td><strong>Obs.</strong></td>
<td>8,443</td>
<td><strong>2,901</strong></td>
</tr>
</tbody>
</table>

**Notes:** In Column 1 the parameters are estimated by GMM on PSID. Standard errors clustered by household in parenthesis. Parameter estimates reported in Column 2 use matched moments from ATUS and CEX data. *, **, *** = Significant at 10%, 5%, and 1%.
Estimated wage process follows from BPS (2016):

- $\sigma^2_{u_1} = 0.0275$, $\sigma^2_{u_2} = 0.0125$, $\sigma^2_{v_1} = 0.0303$, $\sigma^2_{v_2} = 0.0382$,
- cross wage correlations are small and positive, see BPS.
- No insurance here!

Wages of both earners (transitory and permanent) discretized.

Assets discretized, assuming net worth positive constraint.

Discrete unobserved preference heterogeneity/types.
## Comparing Time Use Responses for Low and High Assets Cases

<table>
<thead>
<tr>
<th></th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With kids</td>
<td>With W.o. kids</td>
<td>With kids</td>
<td>With kids</td>
</tr>
<tr>
<td>Low (lowest quartile) assets at age 25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. $\Delta u_1$</td>
<td>-0.15</td>
<td>-0.17</td>
<td>0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\Delta u_2$</td>
<td>0.02</td>
<td>~0</td>
<td>-0.10</td>
<td>-0.12</td>
</tr>
<tr>
<td>Perm. $v_1$</td>
<td>-0.07</td>
<td>-0.06</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>$v_2$</td>
<td>0.10</td>
<td>0.10</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>High (top quartile) assets at age 25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. $\Delta u_1$</td>
<td>-0.22</td>
<td>-0.24</td>
<td>-0.05</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\Delta u_2$</td>
<td>-0.03</td>
<td>-0.06</td>
<td>-0.14</td>
<td>-0.16</td>
</tr>
<tr>
<td>Perm. $v_1$</td>
<td>-0.10</td>
<td>-0.08</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>$v_2$</td>
<td>0.09</td>
<td>0.09</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
</tbody>
</table>
Own elasticity:

\[ \eta_{h_2,w_2} = -\eta_{l_2,w_2} \frac{L_2}{H_2} - \eta_{t_2,w_2} \frac{T_2}{H_2} \]

where we expect \( \eta_{l_2,w_2} < 0, \eta_{t_2,w_2} < 0 \)
Relating Frisch Time Use to Labor Supply Elasticities (Wife’s Example)

- Own elasticity:
  \[ \eta_{h_2,w_2} = -\eta_{l_2,w_2} \frac{L_2}{H_2} - \eta_{t_2,w_2} \frac{T_2}{H_2} \]
  where we expect \( \eta_{l_2,w_2} < 0, \eta_{t_2,w_2} < 0 \)

- Cross elasticity:
  \[ \eta_{h_2,w_1} = -\eta_{l_2,w_1} \frac{L_2}{H_2} - \eta_{t_2,w_1} \frac{T_2}{H_2} \]
  where signs are unrestricted, but:
  - Complementarity of leisure consistent with \( \eta_{l_2,w_1} < 0 \)
  - Specialization in caring for children consistent with \( \eta_{t_2,w_1} > 0 \)
### Comparing Consumption and Hours Responses for Low and High Assets Cases

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>H₁</th>
<th>H₂</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With</td>
<td>W.o.</td>
<td>With</td>
</tr>
<tr>
<td></td>
<td>kids</td>
<td>kids</td>
<td>kids</td>
</tr>
<tr>
<td><strong>Low (lowest quartile) assets at age 25</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans.  Δu₁</td>
<td>0.22</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td>Δu₂</td>
<td>0.21</td>
<td>0.15</td>
<td>-0.04</td>
</tr>
<tr>
<td>Perm.  v₁</td>
<td>0.40</td>
<td>0.42</td>
<td>0.11</td>
</tr>
<tr>
<td>v₂</td>
<td>0.38</td>
<td>0.39</td>
<td>-0.13</td>
</tr>
<tr>
<td><strong>High (top quartile) assets at age 25</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans.  Δu₁</td>
<td>0.07</td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td>Δu₂</td>
<td>0.06</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Perm.  v₁</td>
<td>0.34</td>
<td>0.38</td>
<td>0.18</td>
</tr>
<tr>
<td>v₂</td>
<td>0.34</td>
<td>0.35</td>
<td>-0.16</td>
</tr>
</tbody>
</table>
Decomposing Consumption Smoothing

- Counterfactual consumption response to a male’s permanent wage shock in two key components:
  - insurance via family labour supply, and
  - insurance through savings.

- Wife’s response to husband’s permanent wage:
  - leisure complementarity,
  - specialization,
  - wealth effect.

- We illustrate these channels by decomposing the average simulated counterfactual response to a permanent shock.
**What Does a 10% Permanent Reduction in Husband’s Hourly Wage Look Like?**

<table>
<thead>
<tr>
<th></th>
<th>Husband</th>
<th>Wife</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption:</td>
<td>-4.2%</td>
<td></td>
</tr>
<tr>
<td>After-tax household earnings:</td>
<td>-5.1%</td>
<td></td>
</tr>
<tr>
<td>pre-tax household earnings:</td>
<td>-5.6%</td>
<td></td>
</tr>
<tr>
<td>Earner’s average share of pre-tax earnings:</td>
<td>0.66</td>
<td>0.34</td>
</tr>
<tr>
<td>Earner’s pre-tax earnings response:</td>
<td>-10.4%</td>
<td>+3.3%</td>
</tr>
<tr>
<td>Hours</td>
<td>-1.0%</td>
<td>+4.2%</td>
</tr>
<tr>
<td>Leisure</td>
<td>+1.3%</td>
<td>-1.4%</td>
</tr>
<tr>
<td>Parental time</td>
<td>+0.7%</td>
<td>-5.1%</td>
</tr>
</tbody>
</table>

**Notes:** for a sample of working husbands and wives, working at least 80 hours per year. Based on the regressions run at age 35 in the model.
Mother’s labor supply response to a persistent adverse shock (10%) to husband’s earnings
Mother’s time with children response to a persistent adverse shock to husband’s earnings
### Policy Counterfactuals

<table>
<thead>
<tr>
<th></th>
<th>(1) C</th>
<th>(2) H&lt;sub&gt;1&lt;/sub&gt;</th>
<th>(3) H&lt;sub&gt;2&lt;/sub&gt;</th>
<th>(4) E&lt;sub&gt;2&lt;/sub&gt;</th>
<th>(5) L&lt;sub&gt;1&lt;/sub&gt;</th>
<th>(6) L&lt;sub&gt;2&lt;/sub&gt;</th>
<th>(7) T&lt;sub&gt;1&lt;/sub&gt;</th>
<th>(8) T&lt;sub&gt;2&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A: Exp 1: Unconditional Subsidy for Families with Young Children (yk)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.6%</td>
<td>-0.4%</td>
<td>-0.7%</td>
<td>-0.4%</td>
<td>0.4%</td>
<td>0.3%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before yk</td>
<td>0.9%</td>
<td>-0.4%</td>
<td>-0.5%</td>
<td>-0.2%</td>
<td>0.4%</td>
<td>0.4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With yk</td>
<td>1.3%</td>
<td>-0.6%</td>
<td>-1.8%</td>
<td>-1.0%</td>
<td>0.8%</td>
<td>0.7%</td>
<td>0.2%</td>
<td>1.0%</td>
</tr>
<tr>
<td>After yk</td>
<td>0.1%</td>
<td>-0.1%</td>
<td>-0.1%</td>
<td>-0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption equivalent utility value: 0.95%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **B: Exp 2: Employment Subsidy for Wives with Young Children (yk)** |
| Total | 0.1%  | -0.2%           | 1.9%           | 4.6%           | 0.2%           | -0.5%          |                 |                 |
| Before yk | 0.9%  | -0.4%           | -0.5%           | -0.2%          | 0.4%           | 0.4%           |                 |                 |
| With yk   | -0.3% | -0.3%           | 6.5%           | 13.0%          | 0.3%           | -1.7%          | 0.3%           | -5.7%          |
| After yk  | 0.1%  | -0.1%           | -0.1%           | -0.1%          | 0.1%           | 0.1%           |                 |                 |
| Consumption equivalent utility value: 0.17% |
This research implies that family labor supply can be a key mechanism for ‘insuring’ unexpected shocks especially for younger families and for those with limited access to assets, leisure time turns out to be a Frisch complement but a Marshallian substitute.
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But where do these hours adjustments come from?

Time-use data allowed us to unpack what’s going on.
This research implies that family labor supply can be a key mechanism for ‘insuring’ unexpected shocks especially for younger families and for those with limited access to assets, leisure time turns out to be a Frisch complement but a Marshallian substitute. But where do these hours adjustments come from? Time-use data allowed us to unpack what’s going on. A tension between the desire of spouses to spend leisure time with each other, and the specialization in care of children, complementarity in leisure but specialization in childcare time. family labor supply flips from (Frisch) substitutes to (Frisch) complements as the child ages.
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But where do these hours adjustments come from?

Time-use data allowed us to unpack what’s going on.

A tension between the desire of spouses to spend leisure time with each other, and the specialization in care of children,

- complementarity in leisure but specialization in childcare time.
- family labor supply flips from (Frisch) substitutes to (Frisch) complements as the child ages.

It is mother’s time with children that takes a hit.
SUMMARY AND NEXT STEPS

- Study the interaction between time spent with children, labor supply responses and consumption insurance.
- Combine data on time use, wage, consumption, income, labor supply and assets from the PSID and ATUS.

We find:
- The presence of young children give rises to Frisch substitutability of hours between spouses.
- A switch to Frisch complements as children age and leave home.
- A strong "added-worker" effect as a response to a permanent shock.
- The response of time with children to permanent shocks is important for understanding consumption insurance from labor supply.

Natural next steps:
- study the implications for child outcomes, currently linking to CDS experience/human capital = as in BDMS (Ecta 2016),
- other types of (partially insured) shocks,
- allow for unusual shocks and nonlinear persistence in the wages as in ABB (2017).
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  - study the implications for child outcomes, currently linking to CDS experience/human capital => as in BDMS (Ecta 2016),
  - other types of (partially insured) shocks,
  - allow for unusual shocks and nonlinear persistence in the wages as in ABB (2017).
Nonlinear progressive taxation (including EITC, child tax credits, SNAP and TANF) is approximated by:

\[ T \left( \sum_{j=\{1,2\}} H_{j,t} W_{j,t}; z_t \right) \approx (1 - \chi_t(z_t)) \left( b_t(z_t) + \sum_{j=\{1,2\}} H_{j,t} W_{j,t} \right)^{1-\mu_t(z_t)} \]

\( \chi_t, \mu_t \) and \( b_t \) chosen to match the tax scheme and can depend on year and family composition.

Advantages of this function:

- Performs well for the US tax system (see next slide).
- Allows for extensive margin.
**PERFORMANCE OF THE TAX AND BENEFIT FUNCTION**

Source: Blundell, Pistaferri and Saporta-Eksten, 2016
Transitory vs. Permanent Wage Shock

A transitory wage shock
A permanent wage shock

Age

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Continue with the wife’s cross elasticity:

$$\eta_{h2,w1} = -\eta_{t2,w1} \frac{T^l_2}{H_2} - \eta_{t2,w1} \frac{T^c_2}{H_2}$$

Where do children show up?

- \(\frac{T^l_2}{H_2}, \frac{T^c_2}{H_2}\) (and similarly for the husband \(\frac{T^l_1}{H_1}, \frac{T^c_1}{H_1}\)):
  - The case of no children: \(\frac{T^c_2}{H_2} = 0\). With leisure complementarity: \(\eta_{t2,w1} < 0, \eta_{h2,w1} > 0\).
  - The case of very young children: \(\frac{T^c_2}{H_2} \gg 0\). If \(\eta_{t2,w1} > 0, \eta_{h2,w1}\) might become negative.

Without separability between sub-aggregates, the Frisch elasticities of time use (e.g. \(\eta_{t2,w1}\) and \(\eta_{t2,w1}\)) might depend on children presence and ages.
Labor Supply Elasticities and Children

Continue with the wife’s cross elasticity:

$$\eta_{h_2,w_1} = -\eta_{t_2,w_1} \frac{T_t}{H_2} - \eta_{t_c,w_1} \frac{T_c}{H_2}$$

Where do children show up?

$$\frac{T_t}{H_2}, \frac{T_c}{H_2}$$ (and similarly for the husband $$\frac{T_t}{H_1}, \frac{T_c}{H_1}$$):

- The case of no children: $$\frac{T_c}{H_2} = 0$$. With leisure complementarity:
  $$\eta_{t_2,w_1} < 0, \eta_{h_2,w_1} > 0$$.

- The case of very young children: $$\frac{T_c}{H_2} >> 0$$. If $$\eta_{t_c,w_1} > 0$$, $$\eta_{h_2,w_1}$$ might become negative.

Without separability between sub-aggregates, the Frisch elasticities of time use (e.g. $$\eta_{t_2,w_1}$$ and $$\eta_{t_c,w_1}$$) might depend on children presence and ages.
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<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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<tbody>
<tr>
<td>Hours of work: wife</td>
<td>1,251</td>
<td>1,248</td>
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<td>with young kids</td>
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<td></td>
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<tr>
<td>Hours of work: wife</td>
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<td>1,816</td>
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<td>without young kids</td>
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<td>Hours of work: husband</td>
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<td>2,121</td>
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<td>without young kids</td>
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<td>Hours of parental time:</td>
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<td>778</td>
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<td>wife with young kids</td>
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<td>Hours of parental time:</td>
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<td>337</td>
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<td>husband with young kids</td>
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<tr>
<td>Interquartile range hours: wife</td>
<td>1,818</td>
<td>1,957</td>
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<td>with young kids</td>
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<tr>
<td>Interquartile range hours: wife</td>
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<td>605</td>
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<td>without young kids</td>
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<td>Employment probability of</td>
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<td>0.76</td>
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<td>wife with young kids</td>
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<td>Employment probability of</td>
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<tr>
<td>Change in consumption</td>
<td>0.075</td>
<td>0.073</td>
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<tr>
<td>when kid is born</td>
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</tbody>
</table>

**B. Non-targeted Moment (Wife 50-55, no kids)**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>Hours of work: wife</td>
<td>1,411</td>
<td>1,633</td>
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<tr>
<td>(aged 50-55, no kids)</td>
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<tr>
<td>Hours of work: husband</td>
<td>1,910</td>
<td>2,036</td>
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<td>(aged 50-55, no kids)</td>
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<td>Employment probability of</td>
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<td>0.83</td>
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<tr>
<td>wife (aged 50-55, no kids)</td>
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<tr>
<td>Interquartile range hours of</td>
<td>1,485</td>
<td>1,311</td>
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<tr>
<td>wife (aged 50-55, no kids)</td>
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</tbody>
</table>