Macroeconomics Prelim

Friday, July 15, 2016

Please answer all five questions. The number of points for each question is indicated in parenthesis. If you find a question to be ambiguous, say why, sharpen up the question, and proceed. Please answer each question in a separate blue book. Good Luck!

Question 1 (*30 pts*)

Consider the following economy. There is a continuum of identical infinitely lived households. Each household is endowed with a unit of time and maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - \alpha C_t)^{1-\sigma}}{1-\sigma} - A \frac{h_t^{1+\gamma}}{1+\gamma} \right], \qquad \alpha, \beta \in (0,1), \quad \sigma, A, \gamma > 0.$$

where c_t and h_t are individual consumption and hours worked while C_t is aggregate consumption (taken as given by each household). A unique final good y_t is produced by a large number of competitive firms. These firms use the following constant return to scale technology that requires a continuum of intermediate inputs $y_{it} \in [0,1]$

$$y_t = \left(\int_0^1 y_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}, \quad 0 \leq \eta < 1.$$

Profits of the final good producers are given by

$$y_t - \int_0^1 p_{it} y_{it} di,$$

where p_{it} denotes the price of intermediate good i relative to the the price of the final good. Each intermediate good is produced by a monopolistic competitive firm that uses the following constant returns to scale technology

$$y_{it} = z_t k_{it}^{\theta} h_{it}^{1-\theta},$$

where z_t is an aggregate productivity shock, and k_{it}, h_{it} are capital stock and labor employed by firm i, respectively. Profits of each intermediate firm are given by

$$\pi_{it} = z_t k_{it}^{\theta} h_{it}^{1-\theta} - w_t h_{it} - k_{it} r_t.$$

where w_t and r_t are the wage and the rental rate on capital. The budget constraint of the representative household is given by

$$c_t + k_{t+1} - (1-\delta) k_t = (1-\tau_{ht}) w_t h_t + (1-\tau_{kt}) \left(r_t k_t + \int_0^1 \pi_{it} di \right) + \tau_{kt} \delta k_t + T_t$$

where τ_{kt} and τ_{ht} are capital and labor income taxes and T_t are lump sum transfers (taxes if negative). Assume that the government runs a balanced budget in every period.

a) (*5 pts*) Show that the demand function faced by the intermediate goods producers is

$$y_{it} = p_{it}^{-\frac{1}{\eta}} y_t. \tag{1}$$

b) (*5 pts*) Denote the marginal cost of the intermediate goods firm at time t as λ_t. Show that, conditional on the demand function in the first part of this question, the firm optimally sets its price as a fixed markup over marginal cost.

In addition, show that the marginal cost for the i^{th} intermediate goods firm is

$$\lambda_t = \frac{1}{z_t}\theta^{-\theta}(1-\theta)^{-(1-\theta)} w_t^{1-\theta} r_t^{\theta}. \tag{2}$$

c) (*5 pts*) Consider a symmetric equilibrium, so that $p_{it} = p_{jt}$, $k_{it} = k_{jt}$, $h_{it} = h_{jt}$, $y_{it} = y_{jt}$, $\pi_{it} = \pi_{jt}$ for all i, j. Derive the equilibrium value of p_{it}. Derive an expression relating the equilibrium value of π_{it}/y_{it} to model parameters. Derive the aggregate production function, i.e., the relation between the output of the final good y_t and the average capital and labor in the economy k_t and h_t. Finally, write the resource constraint for this economy.

d) (*5 pts*) Show that in equilibrium, the household's income (i.e., the expression on the right of its budget constraint) is equal to the total production of final goods (this is Walras' Law).

e) (*5 pts*) Write down the equilibrium conditions that characterize a competitive equilibrium for this economy and the first order necessary condition associated with optimality of the planning problem.

Hint: A planner internalizes the aggregate consumption externality in the utility function and ignores the profit maximization condition of the monopolistic competitive firms producing the intermediate goods.

f) (*5 pts*) Derive expressions for τ_{kt} and τ_{ht} which have the consequence that the planner's allocations are implemented in a competitive equilibrium. Show that there are two forces operating on this choice of τ_{ht} : one makes it positive, and the other negative. Provide the underlying intuition. Show that τ_{kt} is not constant.

Question 2 (*20 pts*)

Consider a Lucas tree economy populated by a representative household who is endowed with one tree. The tree produces a stream of dividends, d_t, where $d_0 = 1$. The dividend growth rate, $\frac{d_{t+1}}{d_t}$, can take on one of two values, $\mu + \sigma$ or $\mu - \sigma$, where $\mu > 1$ and $\mu > \sigma > 0$. The dividend growth rate is a Markov chain with transition matrix P. In particular, assume P is a symmetric matrix where the probability of switching growth rates is p, where $p \in (0, 1)$.
The preferences of the household are give by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \ln c_t.$$

At each date, there are markets for consumption goods, trees, and state-contingent claims that pay one unit of consumption tomorrow in a particular state of the world.

a) (*4 pts*) Write the household's problem in recursive formulation. Think carefully about what the household's state variables are.

b) (*4 pts*) Define a recursive competitive equilibrium.

c) (*4 pts*) Solve for the equilibrium pricing function for trees. You should find that the price-dividend ratio for trees is constant over time.

d) (*4 pts*) Solve for the pricing functions for state-contingent claims.

e) (*4 pts*) Add a riskless bond to this economy (a sure claim to one unit of the consumption good next period). Compute the price of a riskless bond.
Hint: There is no need to resolve for the equilibrium price functions.

Question 3 (*20 pts*)

We will use a version of the new Keynesian model with no capital seen in class, to ask how an economy adjusts in the short run to changes in the expected long-run growth rate of productivity. The intra-period utility function is

$$U(C_t, N_t) = \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}.$$

Consumption satisfies the Euler equation

$$C_t^{-1} = \beta R_t E_t \left[C_{t+1}^{-1} \right],$$

where R_t is the real interest rate. The production function is linear in labor

$$Y_t(i) = A_t N_t(i).$$

Productivity A_t grows at the rate g, that is,

$$A_{t+1} = e^g A_t.$$

Our objective is to characterize what happens when productivity is growing for a long time at the constant rate g and a one time, unexpected shock hits at $t = T$, which reduces the growth rate from g to $g' < g$. There are no other shocks.

a) (*3 pts*) Define $\tilde{c}_t = \log \frac{C_t}{A_t}$ and $r_t \equiv \log(\beta R_t)$. Show that the Euler equation can be rewritten as follows.

$$\tilde{c}_{t+1} - \tilde{c}_t = r_t - g. \tag{3}$$

Hint: Recall that there are no shocks, so you can remove the expectation operator and do this step without approximations.

b) (*3 pts*) In a flexible prices allocation the labor supply is constant and so is the ratio C_t/A_t. What restriction does this imply on the real interest rate r_t?

Assume now that prices are sticky and monetary policy is determined in such a way that, in equilibrium, the real interest rate follows the process

$$r_t = r_{t-1} + \alpha(\tilde{c}_t - \tilde{c}^*), \tag{4}$$

where $\tilde{c}^*$ is the constant value of $\tilde{c}_t$ consistent with a flexible price allocation. Notice that here we are making assumptions directly about the real interest rate implied by the policy rule, which is unlike our usual approach of defining monetary policy in terms of the nominal rate. This is done to simplify things. Assume that the economy starts in a steady state with constant productivity growth g, a constant $\tilde{c}_t = \tilde{c}^*$ and the constant real interest rate derived in (b). At $t = T$ agents realize that *from then on* the growth rate of A_t will be $g' < g$. Define

$$\tilde{r}_t \equiv r_t - g'.$$

c) (*4 pts*) Use equation (3) and equation (4) to obtain a dynamic system of difference equations in $\tilde{c}_t, \tilde{r}_t$.

d) (*4 pts*) Using the method of undetermined coefficients conjecture and verify that the solution of the two difference equations in (c) satisfies

$$\tilde{c}_t = \tilde{c}^* - \phi \tilde{r}_{t-1}.$$

for some $\phi \in (0, 1/\alpha)$. What are the dynamics of $\tilde{r}_t$ in this solution? That is, write an equation that determines the value of $\tilde{r}_t$ as a function of $\tilde{r}_{t-1}$.
Hint: no need of giving an explicit solution for ϕ, a graphic representation suffices.

e) (*3 pts*) What is the initial value of $\tilde{r}_{T-1}$ when the shock at T occurs? Is it positive, negative or equal to zero?
Hint: Be careful with the definition of $\tilde{r}$ and the assumptions made about the initial steady state.

f) (*3 pts*) What does $\phi > 0$ implies for answering our original question: what are the short run effects of a long run anticipated change in g? Provide interpretation.

Question 4 (*15 pts*)

Consider an economy populated by a continuum of measure one of consumer who each have preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t).$$

In each period, half of all consumers have endowment $y_t = 1 + \epsilon$, whereas the other half has income $y_t = 1 - \epsilon$, where $0 < \epsilon < 1$. For a given consumer, the income shock is i.i.d. over time, i.e., in each period the probability of getting low or high income is 0.5 each. Risk sharing in this economy is limited by the fact that consumers can withdraw from trade and live in autarky. A consumer who lives in autarky consumers her endowment for two periods. In the third period, a consumer in autarky joins a commune of autarkic agents who practice perfect risk sharing, so that $c_t = 1$ from that time on.

a) (*5 pts*) Find a recursive formulation for the planning problem of a risk-neutral insurance company that can commit to any contract, has access to an outside credit market at interest rate $1 + r = \frac{1}{\beta}$, and would like to minimize the cost of providing reservation utility v to a household, before the current endowment shock is realized.

   Hint: Don't forget to write down the promise keeping and participation constraints, and to provide an expression for the autarky value for both types of households.

b) (*5 pts*) Let us now consider the two-sided lack of commitment problem. Provide a recursive formulation for the problem of a household with endowment y_t that promised reservation utility v to a household with income $2 - y_t$. Note that you need to take into account the participation constraints of both the insured and insurer households.

c) (*5 pts*) Conjecture that equilibrium consumption takes the form

$$c_t = \begin{cases} 1 + \gamma, & \text{if } y_t = 1 + \epsilon, \\ 1 - \gamma, & \text{if } y_t = 1 - \epsilon, \end{cases}$$

   where $0 \leq \gamma \leq \epsilon$. Express the (potentially) binding participation constraint in terms of ϵ and γ. Assume ϵ is large enough such that there exists a level of γ that satisfies the participation constraints. Find what is the optimal contract. i.e., what is the optimal level of γ. You do not need to provide an analytic expression for γ, a graphical argument suffices.

   Hint: The optimal level of γ is the smallest $\gamma \in (0, \epsilon)$ that satisfies the participation constraints.

Question 5 (*15 pts*)

Consider the neoclassical growth model we have covered in class. Final good firms use the production function $y_t = \exp\{z_t\}^{1-\alpha} k_t^{\alpha}$, where z_t is a stochastic technological parameter. The household utility function is of the CRRA type, with the elasticity of intertemporal substitution (EIS) parametrized by σ, $u(c_t) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$. The household discounts future payoffs at the rate β. We assume that z_t follows an univariate Gaussian AR(1) processes,

$$z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t} \qquad \varepsilon_{z,t} \sim \mathcal{N}(0,1),$$

where ρ_z is in $(0,1)$. Assuming that capital depreciate instantaneously ($\delta = 1$), we have that the law of motion for capital and the resource constraint can be expressed as,

$$k_{t+1} = y_t - c_t$$

a) (*5 pts*) Set up recursively the planning problem for this economy. Be careful in specifying the state variables for this decision problem, and their law of motions. Use the first order conditions and the envelope condition to derive the Euler equation.

b) (*5 pts*) After log-linearizing the law of motion for capital and the Euler equation around the deterministic steady state we obtain

$$\alpha\beta \hat{k}_{t+1} = \hat{y}_t - (1-\alpha\beta)\,\hat{c}_t,$$

$$\mathbb{E}_t[\Delta \hat{c}_{t+1}] = \sigma \left\{ \mathbb{E}_t[(1-\alpha)(z_{t+1} - \hat{k}_{t+1})] \right\},$$

where $\hat{x}$ represents variable x expressed in log-deviation from its steady state.

Restricting your attention to policy rules that are linear in the state variables,

$$\hat{c}_t = \eta_{ck}\hat{k}_t + \eta_{cz} z_t \qquad \hat{k}_{t+1} = \eta_{kk}\hat{k}_t + \eta_{kz} z_t,$$

describe the steps that you would follow to solve for $\{\eta_{ck}, \eta_{cz}, \eta_{kk}, \eta_{kz}\}$ using the Method of Undetermined Coefficients.

c) (*5 pts*) Consider now the limit for this economy as $\sigma \to 0$. Show that consumption follows a random walk and that $\eta_{kk} = 1$. Discuss the economics of your results.