2016 Microeconomics Prelim

This exam is divided into three question: one each for 410-1, 410-2 and 410-3.

Please note: EACH QUESTION HAS THE **SAME WEIGHT** IN THE EXAM, REGARDLESS OF THE MAXIMUM POINTS POSSIBLE IN THAT QUESTION.

Each question is divided into parts and sub-parts. Partial credit is awarded as indicated.

You should attempt to solve each question.

410-1 Question

Maximum points possible: 25

Consider a firm selling two products, 1 and 2, which it can produce using production functions $f_1(z_1, z_C)$ and $f_2(z_2, z_C)$ that are continuous, differentiable and strictly increasing in each input. Assume free disposal. The firm can either sell the products separately at prices p_1 and p_2 (this is called unbundling), or it can sell them as a bundle, where one unit of the bundle consists of one unit of each product, and the bundle sells as p_B. Let $p = (p_1, p_2, p_B)$. Input prices are $w = (w_1, w_2, w_C)$. Inputs are denoted by $z = (z_1, z_2, z_C)$ and outputs by q_{U_1}, q_{U_2} in the unbundled case and q_B in the bundled case. We will consider cases where the producer cannot choose whether to bundle or unbundle, and cases where the producer can choose.

1. [4 pts] Let $\pi_B(p_B, w)$ be the profit function when bundling and $\pi_U(p_1, p_2, w)$ be the profit function when unbundling, and $\pi(p_B, p_1, p_2, w)$ be the profit function when the firm can choose. Write out the first two profit functions using f_1 and f_2 and the third using π_B and π_U. Denote the optimal output solutions by $q_B^*(p_B, w), q_{U_i}^*(p_1, p_2, w)$ and $q_i^*(p_B, p_1, p_2, w)$ for $i = 1, 2$.

2. [9 pts] Monotonicity and concavity/convexity:

   (a) Will $\pi(p_B, p_1, p_2, w)$ satisfy the usual convexity/concavity properties of profit functions?

   (b) Will $q_B^*(p_B, w), q_{U_i}^*(p_1, p_2, w)$ and $q_i^*(p_B, p_1, p_2, w)$ satisfy monotonicity in output prices? Specify exactly what is or is not strictly/weakly increasing/decreasing in what price. (Do not add more assumptions.)

   (c) If f_i are both strictly concave is the solution to all three problems unique?

3. [12 pts] Suppose in both problems the solution is unique for all prices. Fix prices $(\bar{p}_B, \bar{p}_1, \bar{p}_2)$ such that $\bar{p}_1 + \bar{p}_2 = \bar{p}_B$

   (a) What can you say about the relationship between $\pi_U(\bar{p}_1, \bar{p}_2, w)$ and $\pi_B(\bar{p}_B, w)$?

   (b) Now assume in addition that $\pi_U(\bar{p}_1, \bar{p}_2, w) = \pi_B(\bar{p}_B, w)$. What can you say about $q_B^*(\bar{p}_B, w)$ vs $q_{U_1}^*(\bar{p}_1, \bar{p}_2, w)$ and $q_{U_2}^*(\bar{p}_1, \bar{p}_2, w)$?

   (c) Continue to assume $\bar{p}_1 + \bar{p}_2 = \bar{p}_B$ and $\pi_U(\bar{p}_1, \bar{p}_2, w) = \pi_B(\bar{p}_B, w)$. Can you say anything about the relationship between a derivative of $\pi_U(\bar{p}_1, \bar{p}_2, w)$ and a derivative of $\pi_B(\bar{p}_B, w)$?

   (d) Continue to assume $\bar{p}_1 + \bar{p}_2 = \bar{p}_B$ and $\pi_U(\bar{p}_1, \bar{p}_2, w) = \pi_B(\bar{p}_B, w)$. What can you say about the relationship between $\frac{\partial q_B^*(\bar{p}, w)}{\partial p_B}$ and $\frac{\partial q_{U_i}^*(\bar{p}, w)}{\partial p_i}$?

410-2 Question

Maximum points possible: 20

Consider an economy under uncertainty with two states, two agents with expected-utility preferences, and a single consumption good. Let $u_i : \mathbb{R}_+ \to \mathbb{R}$, π_i, and $\omega_i \in \mathbb{R}^2_+$ respectively denote agent i's Bernoulli utility over (non-negative) consumption of the single good, the probability she attaches to state 1, and her endowment of the good in each of the two states. For the first two parts of this question, assume that

$$\begin{array}{lll} u_1(x) = \sqrt{x} & \pi_1 = \frac{2}{3} & \omega_1 = (1,0) \\ u_2(x) = x & \pi_2 = \frac{1}{3} & \omega_2 = (0,1). \end{array}$$

(a) 5 point. Find all interior Pareto-efficient allocations.

(b) 5 points. Find all Arrow-Debreu equilibrium allocations.

Now consider the following, alternative notion of Pareto dominance (I state it for an economy with two states, two agents and a single consumption good, such as the one under consideration here):

> Allocation $x \in \mathbb{R}^{2\cdot 2}_+$ *robustly Pareto dominates* allocation $y \in \mathbb{R}^{2\cdot 2}_+$ if (i) it is feasible, (ii) for all $i, j \in \{1, 2\}$,
>
> $$\pi_j u_i(x_{1i}) + (1 - \pi_j)u_i(x_{2i}) \geq \pi_j u_i(y_{1i}) + (1 - \pi_j)u_i(y_{2i}),$$
>
> and (iii) at least one of the inequalities in (ii) is strict.

The key point is that each agent i must like x at least as much as y both using her own belief, and the other agent's belief. This provides a sort of safeguard to each agent—even if her belief is wrong, it must still be the case that x is weakly better using the other agent's belief.

For the next two parts of this question, continue to consider an economy with two states, two agents, a single consumption good, and no aggregate uncertainty. However, now assume arbitrary concave, strictly increasing and continuous utility functions $u_i : \mathbb{R}_+ \to \mathbb{R}$, and arbitrary endowment vectors (provided they sum up to a constant vector, so there is no aggregate uncertainty). Finally, assume that *at least one utility function is **strictly** concave.*

Note that, in part (iii) of the definition of robust Pareto dominance, the strict inequality may occur only for $i \neq j$. Thus, it is not immediate from the above definition that robust Pareto dominance implies standard Pareto dominance. Part (c) of this question asks you to prove that this *is* indeed the case under the above assumptions on preferences. Finally, part (d) relates to the welfare theorems.

(c) 5 points. Show that, if an allocation y is robustly Pareto-dominated by another allocation x, it is also Pareto-dominated in the usual sense by some, possibly different, allocation y'.

(d) 5 points. Show that any Arrow-Debreu equilibrium allocation is robustly Pareto-efficient. Then, show that a robustly Pareto-efficient allocation that is not also Pareto-efficient in the usual sense cannot be supported as an Arrow-Debreu equilibrium (with transfers).

PHILOSOPHICAL BACKGROUND (not required to answer this question): Your finding in (a) implies that interior, full-insurance allocations are *not* Pareto-efficient: they are Pareto-dominated by some other allocation where one consumer gets more consumption in one state, and the other gets more consumption in the other state. That is, such allocations exhibit "betting."

A recent literature takes issue with this conclusion. It puts the blame on the standard definition of Pareto dominance in settings where, as is the case here, agents have different subjective beliefs. The argument, roughly speaking, is that it can't be that both agents are "right" in their beliefs; the fact that both agents agree to a trade may not seem as compelling evidence that the trade is mutually beneficial if (at least) one of them does so due to "wrong" beliefs. (You may or may not buy this argument, by the way. I am just reporting it as background.)

This literature then proposes strengthenings of the notion of Pareto dominance that, in a way, build in some form of robustness to misspecified beliefs. This question explores one such definition. One can show that, in the economy considered above, every full-insurance allocation (i.e., an allocation in which consumption is state-independent) is robustly Pareto-efficient—which was one of the intended purposes of the definition. Part (d) shows that robust Pareto dominance is indeed stronger than standard dominance (so that the corresponding notion of Pareto efficiency is weaker.)

Prelim Question 410-3

Maximum points possible: 25

Consider the following variant of Rubinstein's alternating-offer bargaining game examined in class. Two players take turns proposing a division of a unit surplus. Player 1 is the first player to make a proposal. When an offer is rejected, the game moves to the next period, and the player rejecting the offer in the previous period becomes the proposer. When an offer is accepted, the game ends. Let $t \in \mathbb{N}$ denote the time at which an offer is accepted, $x_i \in [0, 1]$ the share of the surplus received by player $i = 1, 2$ at the time the offer is accepted, and $\delta \in (0, 1)$ the common discount factor. Contrary to Rubinstein's original model, assume flow payoffs take the form

$$u_1(x_1, t) = \begin{cases} \delta^t \left(x_1 - \theta \max\{\alpha - x_2; 0\}\right) & \text{if } t \in 2\mathbb{N}{-}1 \\ \delta^t x_1 \text{ otherwise} \end{cases}$$

$$u_2(x, t) = \begin{cases} \delta^t \left(x_2 - \theta \max\{\alpha - x_1); 0\}\right) & \text{if } t \in 2\mathbb{N} \\ \delta^t x_2 \text{ otherwise} \end{cases}$$

where $\theta, \alpha \in (0, 1)$ are two scalars. The interpretation is the following. In case player i's offer is accepted, player i suffers a disutility equal to $\theta(\alpha - x_j)$ from offering the opponent a share of the surplus $x_j < \alpha$. Such disutility may capture "guilt" stemming from social norms, reputational considerations, or other channels. The scalar θ parametrizes the intensity of such effect. Note that this effect is absent if player i is the respondent. Total payoffs take the familiar discounted time-additive structure as in Rubinstein's original model.

In answering the questions below, write down all the supporting arguments carefully.

(a, 7 points) Suppose $\alpha \leq \delta/(1+\delta)$. Characterize all symmetric stationary SPNE of the above game [Hint: Recall that a symmetric stationary strategy profile is one in which each player $i = 1, 2$ offers the opponent a fraction of the surplus $x_j = R$ when player i is the proposer, and follows the monotone acceptance rule of accepting any share of the surplus $x_i \geq R$ offered by player j and rejecting any offer $x_i < R$, when he is the respondent]. First establish whether the one-stage-deviation principle holds in this environment. If it does, explain how to use it to arrive at the result. If it does not, then explain how to establish the result following an alternative route.

(b, 7 points) Next, suppose $\alpha > \delta/(1 + \delta)$. Characterize all symmetric stationary SPNE of the above game.

(c, 6 points) Now assume that the critical share α such that a player offering the opponent less than α experiences guilt, instead of being exogenous, is part of the equilibrium. In particular, interpret α as the share of the surplus that the respondent expects in equilibrium. Find all values of α such that, when the proposer experiences guilt whenever he offers less than α, then, in the symmetric stationary SPNE of the above game, the proposer offers exactly $R = \alpha$.

(d, 5 points) Next consider the following variant of the payoff specification assumed above. Suppose that there exist $\alpha_j \in (0, 1)$, $j = 1, 2$, such that each player $i \neq j$ experiences a disutility equal to $\theta \max\{\alpha_j - x_j; 0\}$ whenever the opponent receives a share of the surplus $x_j < \alpha_j$, *independently* of whether or not i is the proposer. In other words, suppose that flow payoffs are now given by

$$u_i(x_i, t) = \delta^t \left(x_i - \theta \max\{\alpha_j - x_j; 0\}\right), \; i, j = 1, 2, \; j \neq i.$$

Let $\alpha_1 = 1/(1 + \delta)$ and $\alpha_2 = \delta/(1 + \delta)$ be the shares that the two players receive in the unique SPNE of Rubinstein's original bargaining model (which corresponds to $\theta = 0$). Verify whether or not Rubinstein's unique SPNE remains a SPNE under this alternative specification of "guilt aversion."