Macroeconomics Prelim

Monday, July 10, 2017

Please answer all four questions. The number of points for each question is indicated in parenthesis. If you find a question to be ambiguous, say why, sharpen up the question, and proceed. Please answer each question in a separate blue book. Good Luck!

Question 1 (*30 pts*)

Credible Policy in a Laffer Curve Environment

There is a continuum of identical households, each having the following preferences over consumption, $c \geqslant 0$, hours of work, $0 \leqslant l < \infty$, and economy-wide average government consumption, $G \geqslant 0$:

$$u(c, l, G) = c - \frac{l^2}{2}.$$

The representative household's budget constraint is:

$$c \leqslant (1 - \tau) wl,$$

where τ denotes the labor tax rate and w denotes the wage rate. The representative household takes the wage rate as given.

We suppose that there exists a competitive, representative firm with a production function, $y = al$, where $a > 0$ denotes the marginal product of labor and y denotes output.

Let economy-wide average consumption and labor be denoted by C and L, respectively. The government's budget constraint, expressed in per capita terms, is:

$$G \leqslant \tau wL.$$

The timing of the model is as follows. In the first period, households see w, choose how much labor to supply and the labor market clears. At the start of the second period, the government chooses a value for τ which maximizes the utility of the representative agent, subject to the government budget constraint and the requirement, $G = G^*$, where $G^* > 0$ is known by everyone at the start of the first period. After the government selects a value for τ, the household selects consumption to maximize utility subject to its budget constraint and the choice of l that solves its first period problem. Since the tax rate, τ, does not become known until period 2, the household must make its period 1 labor supply decision based on a 'belief', τ^e, about τ.

1. [*5 pts*] Define a competitive equilibrium for this economy.

2. [*5 pts*] We say that a set of allocations for consumption and employment is technologically feasible if it is consistent with the technology, the resource constraint and time endowment. Show that any G^* is technologically feasible in our economy. Show that existence of equilibrium implies an upper bound for G^*. Provide intuition why there exist levels of G that are technologically feasible, yet not feasible in our market economy.

3. [*5 pts*] Suppose $G^* \geq 0$ and is strictly less than the least upper bound consistent with equilibrium. Show that there are two equilibria for this economy, each with a different value of τ. Show that the equilibrium with the lower value of τ, τ_l, is better in terms of the welfare of the representative household.

The next questions explore what the government can do to ensure that there is only one equilibrium and that the outcome in that equilibrium is $G = G^*$ and $\tau = \tau^l$. We modify the environment slightly. We suppose that, although $G = G^*$ remains a target for the government, it is not absolutely necessary that $G = G^*$.

In the previous questions, we assumed that there is nothing the government can do to affect agents' period 1 belief,τ^e, about τ. We refer to this feature of the government problem as *lack of commitment*. We now assume that the government is able to make a binding commitment in period 1 about what it will do in period 2. But, for such a promise to affect agents' period 1 beliefs, it must be that the promise is *credible*. By this we mean that delivering on its promise is consistent with the government's period 2 budget constraint regardless of what happens in period 1 (i.e., regardless of the value of $\tau^e \in [0,1]$).

The relevant concept of equilibrium is a minor adjustment on the one in the second part of this question, with a suitable adjustment to the statement of the government problem.

4. [*7 pts*] Suppose that at the start of period 1, the government makes a promise to set $\tau = \tau^l$ and $G = G^*$ in period 2. Show that this promise is not credible.

5. [*8 pts*] Suppose that in period 1 the government is able to commit to a credible policy in period 2. Describe a credible policy which has the property that the unique equilibrium outcome is $\tau = \tau^l$ and $G = G^*$. Show that the uniqueness result is not robust to changes in assumption about how G enters the utility function (consider, for example, $u(c, l, G) = (G/G^*)\, c - l^2/2$).

Question 2 (*30 pts*)

Consider a Lucas-tree type economy in which the level of the endowment is independently and identically distributed with support $x_t \in (x\prime, x\prime\prime)$. Agents trade one- and two-period bonds that have prices p_{1t} and p_{2t}, respectively, and return 1 unit of consumption at maturity. In addition, agents can purchase a one-period futures contract for the price f_{1t}. A futures contract purchased at time t states that the owner agrees to give up f_{1t} units of consumption in period $t+1$ for the return of 1 unit of consumption in period $t+2$.

1. [*7 pts*] Assuming standard preferences (i.e. infinitely lived, risk-averse agents with time separable preferences), set up the household's maximization problem as a dynamic programming problem. Think carefully about the relevant state variables in your specification. Derive and interpret the associated necessary optimality conditions.

2. [*4 pts*] Define a recursive competitive equilibrium in this economy.

3. [*7 pts*] Find the equilibrium bond prices p_{1t}, p_{2t} and the price of the future contract f_{1t}. Describe how they depend on the state of the economy x_t.

4. (a) [*4 pts*] Derive an exact relationship between these prices; interpret this relationship. You may (but don't have to) assume that all two period bonds are sold after holding them for one period.

   (b) [*4 pts*] Is it the case that

   $$p_{1t} f_{1t} = p_{2t}.$$

   - After providing a proof of your answer, provide intuition for your answer.

   (c) [*4 pts*] Is it the case that

   $$f_{1t} = E_t[p_{1t+1}]?$$

   - After providing a proof of your answer, provide intuition for your answer.

Question 3 (*20 pts*)

Consider an economy composed by a continuum of measure one of infinitely lived, ex-ante identical people indexed by $i \in [0,1]$ whose preferences given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \theta_t^i \log(c_t^i) \right\},$$

where $\beta \in (0,1)$. Each consumer has a fixed endowment of $y = 1$ in every period. The preference shock θ_t^i captures the consumption needs of consumers. In the initial period, all consumers start out with $\theta_0^i = 1$. Subsequently, each period consumers face a constant probability $\phi < 1$ of developing higher consumption needs of $\theta_t^i = 2$ (you can think of the shock as a health shock or a family size shock, i.e., the arrival of a new family member with own consumption needs). Developing higher needs is an absorbing state: once $\theta_t^i = 2$, we have $\theta_{t+1}^i = \theta_t^i = 2$ forever. At the aggregate level, ϕ corresponds to the fraction of the remaining individuals with $\theta_{t-1}^i = 1$ who switch to $\theta_t^i = 2$. There is an aggregate storage technology whereby storing S_{t+1} at time t yields $(1+r)S_{t+1}$ units of consumption at time $t+1$, where $1 + r = 1/\beta$. Consumers start with $S_0 = 0$.

1. [*4 pts*] Define an Arrow-Debreu complete-markets equilibrium (i.e., an equilibrium where each consumer faces a single time-zero budget constraint). Think carefully about the relevant states in your definition. You can deal with the storage technology by setting up a profit-maximization problem for a single competitive (price-taking) firm that operates the storage technology.

2. [*7 pts*] Find the equilibrium. Guidance: First explain why can you solve the central planner problem in order to find the competitive equilibrium allocation. Then solve the central planner's problem in two stages:

   (a) First solve for the intra-temporal allocation of consumption between the different individuals in the economy and express the consumption of each individual in terms of the consumption of the $\theta = 1$ type.

   (b) Then given the expression you found for consumption solve for the optimal path of storage and the level of consumption of $\theta = 1$ type: Use the FOC to show that the consumption level is constant over time. Iterate forward the resource constraint faced by the social planner to get a single time zero resource constraint. Use the time zero resource constraint to solve for the optimal consumption level in terms of parameters of the model. Finally find a recursive formulation for the optimal path of storage.

Once you have the optimal allocation find the equilibrium prices using the individual household problem and the storage firm's problem (if you chose to define one).

3. [*4 pts*] Define a recursive incomplete markets equilibrium for a setting where it is not possible to write contracts contingent on θ_t, but the storage technology is still available. You can treat storage either as an aggregate technology operated by a firm (as above), or assume that each consumer operates the technology independently (you will get the same results either way).

4. [*5 pts*] Define the recursive problem of an individula that already received a high preferences shock i.e $\theta^i = 2$ and of an individual that did not receive a high preference shock yet i.e $\theta^i = 1$ and derive their optimality conditions.

Question 4 (*20 pts*)

Consider the following system of equations

$$\begin{aligned}\mathbb{E}_t[\Delta\hat{c}_{t+1}] &= \hat{\beta}_t \\ \hat{c}_t &= \hat{n}_t + \hat{a}_t, \\ \hat{a}_t &= \phi_a\hat{a}_{t-1} + \varepsilon_{a,t} \\ \hat{\beta}_t &= \phi_\beta\hat{\beta}_{t-1} + \varepsilon_{\beta,t}\end{aligned}$$

where $\hat{c}_t$ and $\hat{n}_t$ represents the log-deviation from steady state of consumption and labor (respectively), $\hat{a}_t$ represents a technology shock, and $\hat{\beta}_t$ is a shock to the rate of time preferences of households.

To place it in context, this system represents the log-linearized equilibrium conditions of a New Keynesian model without capital, with perfectly sticky prices, and with perfect inflation targeting for the monetary authority, similar to the one we studied in class. The first equation represents the Euler equation of households, the second equation the resource constraint, and the last two equations the law of motion for the shocks. Assume that $\varepsilon_{a,t}$ and $\varepsilon_{\beta,t}$ are independent Gaussian i.i.d. processes.

A solution to this system consists of law of motions for the endogenous variables, $\hat{c}_t$ and $\hat{n}_t$, that are linear in the state variables $\{\hat{a}_t, \hat{\beta}_t\}$,

$$\begin{aligned}\hat{c}_t &= \eta_{ca}\hat{a}_t + \eta_{c\beta}\hat{\beta}_t, \\ \hat{n}_t &= \eta_{na}\hat{a}_t + \eta_{n\beta}\hat{\beta}_t.\end{aligned}$$

1. [*5 pts*] Assume that $|\phi_a| < 1$ and $|\phi_\beta| < 1$. Use the method of undertermined coefficients to show that

$$\begin{aligned}\eta_{na} = -1 \qquad & \eta_{n\beta} = \tfrac{1}{(\phi_b - 1)} \\ \eta_{ca} = 0 \qquad & \eta_{c\beta} = \tfrac{1}{(\phi_b - 1)}\end{aligned}$$

2. [*5 pts*] Plot the IRFs of consumption and labor to a technology shock. Discuss the economic intuition for your results. In a separate graph, plot the response of these two variables to the preference shock.

3. [*5 pts*] Show that $\{\hat{n}_t, \hat{c}_t\}$ follows the VAR(1) process

$$\begin{bmatrix} \hat{n}_t \\ \hat{c}_t \end{bmatrix} = \begin{bmatrix} \phi_a & (\phi_\beta - \phi_a) \\ 0 & \phi_\beta \end{bmatrix} \begin{bmatrix} \hat{n}_{t-1} \\ \hat{c}_{t-1} \end{bmatrix} + \begin{bmatrix} -1 & \frac{1}{(\phi_\beta - 1)} \\ 0 & \frac{1}{(\phi_\beta - 1)} \end{bmatrix} \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{\beta,t} \end{bmatrix}.$$

4. [*5 pts*] Suppose that an econometrician wants to identify the dynamic effects of the two shocks using a structural VAR analysis. Propose an identification scheme that would help identifying the two structural shocks. Please describe in details the steps required to perform such analysis.