Econometrics Preliminary Exam
July, 2017

Read the instructions below before starting the exam:

- The preliminary exam consists of **three parts** (one for each quarter of Econ 480).
- You have 180 minutes (1 hour for each part) to solve this exam.
- This is a closed book and notes exam.
- You can use one page (both sides) of hand-written or typed notes.
- Remember to write your name on each examination booklet and on the space provided below.
- You may use a calculator if needed
- Please keep your written answers brief; be clear and to the point.
- We recommend that you circle or otherwise indicate your final answers.
- There is no choice, so remember to answer all questions.
- Good luck!

PLEASE WRITE EACH PART OF THE EXAM IN
A SEPARATE BLUE BOOK.

Name (First Last): ______________________________

Part I (one hour, 60 points)

Consider a population J of travelers, each member of which chooses between two travel modes, labeled 0 and 1. The utility of mode c to person j is $u_{jc\gamma} = \alpha x_{jc\gamma} + \beta z_{jc}$. Here x is travel time, z is travel cost, (α, β) are parameters, and γ denotes the degree of travel congestion. The presence of a γ-subscript on $x_{jc\gamma}$ indicates that travel time may vary with γ. The absence of a γ-subscript on z_{jc} indicates that travel cost does not vary with the degree of congestion.

Assume that travelers do not know γ when they make their decisions. Assume person j chooses a mode that maximizes subjective expected utility $\int u_{jc\gamma} dQ_{j\gamma}$, where $Q_{j\gamma}$ is the subjective distribution that j places on γ. The expected utility of mode c to person j is $\alpha v_{jc} + \beta z_{jc}$, where $v_{jc} \equiv \int x_{jc\gamma} dQ_{j\gamma}$. Let d_j denote the travel mode chosen by person j.

A researcher wants to learn the values of (α, β). He knows a priori that $\alpha < 0$ and $\beta < 0$. For each person $j \in J$, the researcher observes (d_j, z_{j0}, z_{j1}). The researcher does not observe (v_{j0}, v_{j1}), but he can place credible bounds on their values. Specifically, he knows that $(v_{j0}, v_{j1}) \in [v_{j0L}, v_{j0U}] \times [v_{j1L}, v_{j1U}]$, where L and U denote the lower and upper bounds.

A (12 points). What is the identification region for (α, β)? Be as explicit as possible.

B (12 points). Is the assumed model of expected utility maximization refutable? If so, give the conditions in which it is refuted. If not, explain why the model is not refutable.

C (12 points). Is the assumption that each person j has utility function $u_{jc\gamma} = \alpha x_{jc\gamma} + \beta z_{jc}$ refutable? If so, give the conditions in which it is refuted. If not, explain why the assumption is not refutable.

D (12 points). Suppose that, instead of observing (d_j, z_{j0}, z_{j1}) for every person in the population, the researcher observes these values for a sample of N persons. The researcher does not know the process generating the sample; for example, it need not be a random sample. What do the observed data reveal about the identification region for (α, β)?

E (12 points). Suppose the researcher believes that persons do not have complete subjective distributions on γ and, hence, cannot maximize expected utility. Instead, the researcher assumes that each person j knows the bounds $(v_{j0}, v_{j1}) \in [v_{j0L}, v_{j0U}] \times [v_{j1L}, v_{j1U}]$ and chooses a mode that maximizes minimum expected utility. Now what is the identification region for (α, β)? Be as explicit as possible.

Prelim Question, July 2017

Do all problems, show all work, and define any symbols you use that are different from those in the problem statements.

Problem 1 (20 points):

Let the scalar variables Y and X^* be related by

$$Y = \beta X^* + U,$$

where $E(X^*) = 0$, $E(X^{*2}) = \sigma_{X^*}^2 < \infty$, U is independent of X^*, $E(U) = 0$, and $E(U^2) = \sigma_U^2 < \infty$. X^* is not observed. Instead, you observe X, where

$$X = X^* + \varepsilon,$$

ε is independent of X^* and U, $E(\varepsilon) = 0$, and $E(\varepsilon^2) = \sigma_\varepsilon^2$.

a. Suppose there are two independent observations of X for each observation of Y. That is, the data are the random sample $\{Y_i, X_{i1}, X_{i2} : i = 1,...,n\}$, where

$$X_{ij} = X_i^* + \varepsilon_{ij};\ (j = 1,2),$$

X_i^* is the true but unobserved value of X^* corresponding to Y_i, $E(\varepsilon_{ij}) = 0$, $E(\varepsilon_{ij}^2) = \sigma_\varepsilon^2$, and ε_{i1} and ε_{i2} are independent of each other and of X_i^* and U_i. Can you estimate β consistently? If yes, display an estimator and prove that it is consistent.

b. Suppose there is only one observation of X for each observation of Y. Thus, the data are the random sample $\{Y_i, X_i : i = 1,...,n\}$. Define

$$\hat{\beta} = \frac{\sum_{i=1}^n Y_i^2 X_i}{\sum_{i=1}^n Y_i X_i^2}.$$

Under what conditions, if any, is $\hat{\beta}$ consistent for β? Prove your result.

Problem 2 (40 points)

Let the scalar parameter β be estimated by

$$\hat{\beta} = \arg\min_{b \in B} Q_n(b),$$

where B is a parameter set, Q_n is a random function, and n is the sample size. Assume that

i. β is an interior point of B, and $n^{1/2}(\hat{\beta}-\beta) \to^d N(0,\sigma_\beta^2)$ for some $\sigma_\beta^2 < \infty$.

ii. Q_n is twice continuously differentiable, and for any sequence $\{b_n\}$ such that $b_n \to \beta$ as $n \to \infty$

$$\frac{1}{n}\frac{d^2 Q_n(b_n)}{db^2} \to a$$

for some finite, non-stochastic constant a.

iii. σ_β^2 and a are known.

You want to test the hypothesis $H_0: \ \beta = \beta_0$, where β_0 is an interior point of B.

a. If H_0 is true, then $Q_n(\beta_0) - Q_n(\hat{\beta})$ differs from zero only due to random sampling error in $\hat{\beta}$. H_0 is rejected if $Q_n(\beta_0) - Q_n(\hat{\beta})$ is too large. Derive a test statistic that implements this idea and is asymptotically distributed as χ^2 with one degree of freedom if H_0 is true.

b. Is there a function Q_n for which your statistic does not depend on σ_β^2 and a? If so, what is that function? Prove your result.

c. If H_0 is true, then $dQ_n(\beta_0)/db$ differs from zero only due to random sampling error in Q_n. H_0 is rejected if $dQ_n(\beta_0)/db$ is too different from zero. Derive a test statistic that implements this idea and is asymptotically distributed as χ^2 with one degree of freedom if H_0 is true.

d. What are the asymptotic distributions of the statistics in parts a and c under the sequence of local alternative hypotheses $H_1: \ \beta = \beta_0 + n^{-1/2}\Delta$? Present the derivation. Do not just present the result. Assume that (i)-(iii) hold under H_1.

Econometrics Preliminary Exam
Part III (60 points) - July, 2017

Question 1 30

Let (Y, X, Z_1, Z_2, U) be a random vector where all variables take values in $\mathbf{R}$ and have finite first and second moments. Suppose that the causal model for Y states that, for $\beta = (\beta_0, \beta_1)' \in \mathbf{R}^2$,

$$Y = \beta_0 + \beta_1 X + U \ ,$$

where it is known that $U|Z_j \sim N(0, \sigma^2)$ for $j = \{1, 2\}$.

(a) (10 points) Ignore Z_2 for the moment and assume you only observe (Y, X, Z_1). Show that β_1 is identified, clearly stating all conditions required for your result to go through.

(b) (10 points) Now we wish to exploit the presence of Z_2. Assume then that you have a random sample of n i.i.d. observations from the distribution of (Y, X, Z_1, Z_2) and that $E[XZ_2] > E[XZ_1] > 0$. Consider the following estimator of β_1,

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (Z_{1,i} - \frac{1}{n}\sum_{i=1}^n Z_{1,i})(Y_i - \frac{1}{n}\sum_{i=1}^n Y_i Z_{2,i})}{\sum_{i=1}^n (Z_{1,i} - \frac{1}{n}\sum_{i=1}^n Z_{1,i})(X_i - \frac{1}{n}\sum_{i=1}^n X_i Z_{2,i})} \ . \tag{1}$$

Show that $\sqrt{n}(\hat{\beta}_1 - \beta_1) \xrightarrow{d} N(0, \mathbb{V})$ with $\mathbb{V} = \frac{\text{Var}[Z_1]}{(E[X(Z_1 - E[Z_1])])^2}\sigma^2$.

(c) (10 points) Consider an alternative approach to exploit Z_2, involving the following two steps:

- **Step 1.** Project Y on Z_1 (and a constant) and let $\hat{Y}_i$ be the predicted value of Y from this projection. Project X on Z_1 (and a constant) and let $\hat{X}_i$ be the predicted value of X from this projection.
- **Step 2.** Regress $\hat{Y}_i$ on $\hat{X}_i$ (and a constant) using Z_2 as an instrument for $\hat{X}_i$. This leads to the following estimator of β_1,

$$\hat{\beta}_1^* = \frac{\sum_{i=1}^n (Z_{2,i} - \frac{1}{n}\sum_{i=1}^n Z_{2,i})\hat{Y}_i}{\sum_{i=1}^n (Z_{2,i} - \frac{1}{n}\sum_{i=1}^n Z_{2,i})\hat{X}_i} \ . \tag{2}$$

Show that $\sqrt{n}(\hat{\beta}_1^* - \beta_1) \xrightarrow{d} N(0, \mathbb{V}^*)$ and find an expression for $\mathbb{V}^*$.

Question 2 30

Consider the following Difference-in-Differences setup with $\mathcal{J}_1 = \{1\}$ treated groups, $\mathcal{J}_0$ controls groups, $\mathcal{T}_0 = \{1\}$ untreated periods, and $\mathcal{T}_1 = \{2\}$ treated periods. Outcomes are observed for two groups for two time periods. One of the groups (group 1) is exposed to a treatment in period 2 but not in period 1. The second groups is not exposed to the treatment. To be specific, let

$$\{(Y_{j,t}, D_{j,t}) : j \in \mathcal{J}_0 \cup \mathcal{J}_1 \text{ and } t \in \{1, 2\}\}$$

denote the observed data, where $Y_{j,t}$ and $D_{j,t} \in \{0, 1\}$ denote the outcome and treatment status of group j at time t. Assume further that

$$Y_{j,t}(0) = \lambda_t \eta_j + \gamma_t + U_{j,t} \ ,$$

for all (j, t), where $E[U_{j,t}] = 0$, and η_j, γ_t, and λ_t are (non-random) group, time effects, and factor loadings, respectively. The parameter of interest is

$$\theta = E[Y_{1,2}(1) - Y_{1,2}(0)] \ .$$

(a) (10 points) Two natural approaches to identify θ are: (a.1) compare $Y_{1,2}$ with $Y_{j,2}$ for some $j \in \mathcal{J}_0$, and (a.2) compare $Y_{1,2}$ with $Y_{1,1}$. Explain clearly why each of these two approaches do not identify θ without additional assumptions.

(b) (10 points) Assume $\mathcal{J}_0 = \{2\}$ and show whether a difference-in-differences approach identifies θ under the above assumptions. If the answer is yes, explain what is the fundamental assumption that delivers identification. If the answer is no, explain what is the fundamental missing assumption that does not permit identification.

(c) (10 points) Suppose that there exist non-random weights $\{w_j^* : j \in \mathcal{J}_0, w_j^* \geq 0, \sum_{j \in \mathcal{J}_0} w_j^* = 1\}$ such that

$$Y_{1,1} = \sum_{j \in \mathcal{J}_0} w_j^* Y_{j,1} \ . \tag{10}$$

Now let

$$\tilde{Y}_{1,2}(0) = \sum_{j \in \mathcal{J}_0} w_j^* Y_{j,2}$$

be a synthetic control for $Y_{1,2}(0)$. Show whether a synthetic control approach identifies θ under the above assumptions. If the answer is yes, explain what is the fundamental assumption that delivers identification. If the answer is no, explain what is the fundamental missing assumption that does not permit identification.