Macroeconomics Prelim

Wednesday, July 25, 2018

Please answer all four questions. The number of points for each question is indicated in parenthesis. If you find a question to be ambiguous, say why, sharpen up the question, and proceed. Please answer each question in a separate blue book. Good Luck!

Question 1 (*30 pts*)

Consider the following date $t=0$ household problem:

$$V = \max_{\{c_t, l_t, m_t, b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t), \ \beta \in (0,1) \tag{1}$$

$$s.t. \quad m_t + b_t = W_t l_t + m_{t-1} - P_{t-1}c_{t-1} + R_{t-1}b_{t-1} + T_t \tag{2}$$

$$P_t c_t \le m_t \tag{3}$$

$$(m_{-1} - P_{-1}c_{-1} + R_{-1}b_{-1}) \text{ given,}$$

$$0 \le l_t \le N, c_t \ge 0. \tag{4}$$

Here, c_t and l_t denote household consumption and employment. Also, m_t and b_t denote the stock of money and bonds held by the household at the end of period t, and W_t, R_t denote the nominal wage and interest rate, respectively. The condition, (3), corresponds to the requirement that the household have enough cash balances at the end of the period to cover that period's consumption expenditures. The object, T_t, denotes lump sum payments. The household takes $P_t > 0, T_t, R_t \ge 1, W_t > 0$ as given sequences, $t \ge 0$. Output and market clearing in the economy are given by $c_t = y_t = l_t$. The utility function, U, is strictly concave and differentiable and you may suppose that solutions are always interior:

$$c_t > 0, 0 < l_t < N.$$

Suppose the following condition is satisfied:

$$\sum_{t=0}^{\infty} q_t \left[W_t N + T_t\right] < \infty, \tag{5}$$

where

$$q_t = \begin{cases} 1 & t = 0 \\ \frac{1}{R_0 R_1 \cdots R_{t-1}} & t \ge 1 \end{cases}. \tag{6}$$

1. (2 points) To better understand the household constraints, it is convenient to express (2) in a simpler form. Let end of period t assets net of current consumption obligations be denoted by a_t :

$$a_t = b_t + m_t - P_t c_t.$$

Let i_t denote the household's 'income', which is what they can spend if they consume no leisure:

$$i_t = W_t N + T_t.$$

Let s_t denote household 'spending':

$$s_t = P_t c_t + W_t (N - l_t) + (R_{t-1} - 1)(m_{t-1} - P_{t-1} c_{t-1}).$$

Provide intuition about why it makes sense to call s_t 'spending' and show that (2) can be written as follows:

$$a_t = i_t - s_t + R_{t-1} a_{t-1}.$$

2. (3 points) Consider the following lower bound on a_t:

$$a_t \geq -\frac{1}{q_t} \sum_{j=1}^{\infty} q_{t+j} i_{t+j}, \tag{7}$$

for each $t \geq 0$. Show that the object on the right of the inequality is precisely amount of debt that the household could have in period t that it could just barely pay back by setting, $s_{t+j} = 0$, for $t + j$, $j \geq 1$. This is called the *natural debt limit.*

3. (6 points) Show that, given condition (2), the natural debt limit is equivalent to

$$\lim_{T \to \infty} q_T a_T \geq 0 \tag{8}$$

$$\frac{1}{q_t} \sum_{j=1}^{\infty} q_{t+j} i_{t+j} + a_t \geq \frac{1}{q_t} \sum_{j=1}^{\infty} q_{t+j} s_{t+j}. \tag{9}$$

4. (4 points) Derive the first order conditions for labor and for bonds and show that

$$q_t = \frac{\beta^t u_{c,t} P_0}{u_{c,0} P_t}.$$

5. (6 points) The canonical form in Stokey-Lucas for an optimization problem is as follows:

$$\max_{\text{feasible } \{x_t\}} \sum_{t=0}^{\infty} \beta^t F_t (x_{t-1}, x_t).$$

   (a) Show that the household problem in this question can be written in this way. Hint: consider $x_t = (c_t, m_t, b_t)'$. Be sure to define feasibility.

   (b) Show that the household's cash constraint, (3), can be expressed as the requirement,

$$A_t' x_t \geq 0,$$

   and derive the column vector, A_t.

   (c) The natural debt limit, equivalent to (8), is expressed as:

$$\lim_{T \to \infty} q_T a_T = \lim_{T \to \infty} q_T \tilde{A}_T' x_T \geq 0,$$

   and derive the column vector, $\tilde{A}_t$

(d) Show that

$$-\frac{P_0}{u_{c,0}}\beta^T F_{T,2}\left(x_{T-1}, x_T\right) x_T = q_T a_T,$$

where $F_{T,2}$ denotes the rowvector formed by differentiating F_T with respect to x_T.

6. (9 points) Write out the Stokey-Lucas representation of the household problem in Lagrangian form, with the multiplier, $\mu_t \geq 0$, on the cash constraint.

(a) Derive the Euler equations of this representation of the problem.

(b) The transversality condition is

$$\lim_{T\to\infty} \beta^T F_{T,2}\left(x_{T-1}, x_T\right) x_T = 0. \tag{10}$$

Show that if you have a sequence, x_t^*, $t = 0, 1, ...$ which satisfies the Euler equations and the transversality condition, then that sequence generates a return for the household that is no less than the return generated by any other feasible sequence.

Question 2 (*20 pts*)

Consider the planner's problem for a real business cycle model with inelastic labor supply and no trend growth. The representative consumer's preferences are given by

$$E\sum_{t=0}^{\infty}\beta^t ln(C_t) \tag{11}$$

Here $0 < \beta < 1$, C_t denotes time t consumption and E_0 denotes time zero conditional expectations operator.

Output, Y_t, is produced using capital K_t according to the technology

$$Y_t = A_t K_t^{\alpha} \tag{12}$$

where K_t is the capital stock at the start of time t, A_t is a stationary technology shock that evolves according to an AR(1) process. There are adjustment costs to capital, which evolves according to the following production function

$$K_{t+1} = K_t^{\delta} I_t^{1-\delta}. \tag{13}$$

When $\delta = 0$, this looks like a standard growth model with full depreciation, i.e. $K_{t+1} = I_t$. We will focus on the general case where $\delta \in [0, 1]$. The resource constraint is

$$Y_t = C_t + I_t. \tag{14}$$

Write down the recursive formulation of the planner's problem. Use two constraints, the resource constraint, (11), and the capital production, (13). Denote the Lagrange multiplier on the resource constraint as λ_t and the one on the capital production constraint as $\lambda_t q_t$.

1. (5 points) Derive the first order conditions. Provide an Intuitive explanation for q_t.

2. (9 points) Using a guess and verify procedure, solve the model and find the policy functions for K_{t+1}, I_t and C_t. Solve explicitly for these variables as a function of the parameters of the model, A_t and K_t. Hint: start by guessing that investment and consumption are a constant share of of output.

3. (6 points) Explain why we can interpret δ as a parameter that affects the degree of capital adjustment costs in the model. Briefly discuss how and why the response of consumption and investment to technology shocks (A_t) vary with δ.

Question 3 (*20 pts*)

Consider an infinite horizon economy with 2 groups of agents of equal mass, denoted $i = 1, 2$. Agents have preferences represented by the utility function

$$\sum \beta_i^t \left[\log c_{it} + \log\left(1 - n_{it}\right)\right],$$

where c_{it} is consumption and n_{it} is labor supply. The difference between the two groups $i = 1, 2$ is only that

$$\beta_1 < \beta_2,$$

so agents in group 1 are less patient. Competitive firms hire workers and produce using the linear technology

$$Y_t = AN_t,$$

where A is a constant productivity parameter, N_t is aggregate labor supply and Y_t is aggregate output. There is no capital. Prices are flexible. Agents only trade goods and a risk free, one period bond, so their budget constraint is

$$c_{it} + q_t a_{it+1} = w_t n_{it} + a_{it},$$

where w_t is the real wage and q_t is the price of a bond. Agents face the exogenous borrowing limit

$$a_{it} \geq -\phi.$$

The risk free bond is in zero net supply in the economy, so in equilibrium we need $\sum_i a_{it} = 0$. There is no uncertainty (later we'll add an unexpected one-time shock).

1. (2 points) What is the real wage in equilibrium?

2. (3 points) Setup the individual optimization problem for a given sequence of bond prices $\{q_t\}$ and derive optimality conditions.

3. (3 points) Define the net savings

   $$z_{it} \equiv q_t a_{it+1} - a_{it}.$$

   Using optimality conditions from (1) and the budget constraint express c_{it} and n_{it} as functions solely of z_{it}.

4. (4 points) Show that the individual optimization problem can be solved in two stages: first, solving a dynamic problem that gives us an optimal policy $z_{it} = Z_t\left(a_{it}\right)$ (formulate the problem explicitly) and then deriving implications for c_{it} and n_{it}.

5. (4 points) Using the result in (3) and market clearing in the bond market show that aggregate consumption and aggregate labor supply must be constant in equilibrium and independent of the borrowing limit ϕ and of the initial positions $(a_{i0})_{i=1,2}$.

6. (4 points) Suppose that the economy is in a steady state with constant positions a_{it}. Conjecture how the economy will adjust to an unexpected shock that reduces ϕ permanently to a lower level (keep in mind result (4)!)

Question 4 (*30 pts*)

Consider an economy populated by a continuum of measure one of consumers. In each period, half of all consumers have endowment $y_t = 1 + \epsilon$, whereas the other half has income $y_t = 1 - \epsilon$, where $0 < \epsilon < 1$. For a given consumer, the income shock is i.i.d. over time, i.e., in each period the probability of getting low or high income is 0.5 each. In the aggregate, half of consumers have each income shock in every period. The preferences are defined by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where the felicity is:

$$U(c_t) = \begin{cases} c_t & \text{if} \quad c_t \leq 1, \\ \log(c_t) + 1 & \text{if} \quad c_t > 1. \end{cases}$$

The consumer also has to respect a non-negativity constraint on consumption, $c_t \geq 0$.

1. (7 points) Assume that income shocks are observable and that agents can commit. Define and solve for a complete markets competitive equilibrium.

2. (5 points) Define an incomplete-markets equilibrium in which the only asset available is a risk-free bond a_t that is in zero net supply and that trades at interest rate r_t. People are subject to the natural borrowing limit.

3. (5 points) Consider, in partial equilibrium, the savings problem of a consumer in the incomplete-markets setting under the assumption that the interest rate satisfies $1 + r_t = 1/\beta$. Does the result that consumption will approach infinity over time (which we derived with standard preferences) still go through? Write down the usual argument and point out steps (if there are any) that do not go through in this setting.

4. (8 points) Now assume that income shocks are private information. There is a risk-neutral planner who would like to minimize the cost of providing reservation utility w_0 to each agent, and who has preferences:

$$\sum_{t=0}^{\infty} \beta^t z_t.$$

Implicitly, we assume here that the planner has access to an outside credit market and is not subject to an economywide resource constraint. Provide a recursive formulation for the cost minimization problem of the planner, using a scalar utility promise w as the state variable. Regarding the truth-telling constraints, you can assume that only local downward constraints need to be imposed.

5. (5 points) With standard preference, we have derived the result that under the optimal contract, with probability one consumption of a given agent converges to zero (immiseration). Is this still true in our setting? Write down the usual argument and point out steps (if there are any) that do not go through in this setting. *(Hint: Consider an person whose current utility promise* w *satisfies* $w \leq \frac{1}{(1+\epsilon)(1+\beta)}$*. Guess a cost-minimizing way of delivering this utility.)*