Econometrics Preliminary Exam
July, 2018

Read the instructions below before starting the exam:

- The preliminary exam consists of **three parts** (one for each quarter of Econ 480).
- You have 180 minutes (1 hour for each part) to solve this exam.
- This is a closed book and notes exam.
- You can use one page (both sides) of hand-written or typed notes.
- Remember to write your name on each examination booklet and on the space provided below.
- You may use a calculator if needed
- Please keep your written answers brief; be clear and to the point.
- We recommend that you circle or otherwise indicate your final answers.
- There is no choice, so remember to answer all questions.
- Good luck!

PLEASE WRITE EACH PART OF THE EXAM IN
A SEPARATE BLUE BOOK.

Name (First Last): ______________________________

2018 Econometrics Prelim, Part I (one hour, 60 points)

Each member of population J is at risk of opioid addiction. Let each $j \in J$ be offered a new treatment that may prevent addiction in some persons but encourage it in others. Let $z_j = 1$ if person j accepts the offer of the treatment and $z_j = 0$ otherwise. Let y_j be the addiction outcome, with $y_j = 1$ if j becomes addicted and $y_j = 0$ otherwise. It is observed that 70 percent of the population choose to accept the offer and 30 percent choose not to accept it. Among those who accept the offer, 50 percent become addicted. Among those who do not accept the offer, 40 percent become addicted.

Let $t = 1$ denote use of the treatment and $t = 0$ otherwise. Let $y(t) = 1$ if a person with treatment use value t becomes addicted; $y(t) = 0$ otherwise. Treatment response is individualistic; that is, treatment of one person does not affect the outcomes of others. Let $P[y(1) = 1]$ denote the fraction of the population who would become addicted if use of the treatment were mandatory. Let $P[y(0) = 1]$ be the fraction who would become addicted if use of the treatment were prohibited.

A. (12 points) A researcher has the stated empirical evidence but has no other information. What can the researcher conclude about the magnitude of the *relative risk* comparing mandatory treatment and prohibition, defined by the ratio $P[y(1) = 1]/P[y(0) = 1]$?

B. (12 points) Suppose that the researcher assumes each person j knows his or her personal values of $[y_j(0), y_j(1)]$. The researcher also assumes that each person prefers not to become addicted and chooses z_j accordingly; thus, the choice minimizes $[y_j(0), y_j(1)]$. Using the empirical evidence and these assumptions, what can the researcher conclude about the relative risk of addiction?

C. (12 points) Instead of the assumptions made in part B, suppose that the researcher assumes use of the treatment may prevent addiction but never encourages it; thus, $y_j(1) \leq y_j(0)$ for all persons j. Using the empirical evidence and this assumption, what can the researcher conclude about the relative risk of addiction?

D. (12 points) Suppose as in part A that the researcher has the empirical evidence but no other information. What can the researcher conclude about $P[y(0) = y(1) = 1]$; that is, the fraction of the population who become addicted whether or not they receive the treatment?

E. (12 points) Suppose that the researcher does not observe the chosen treatments and outcomes of the entire population. Instead, he observes the values for a random sample of size $N = 100$. Let

$$R_N \equiv P_N[y(1) = 1 \mid z = 1]/P_N[y(0) = 1 \mid z = 0]$$

denote the sample relative prevalence of addiction, comparing the sample members who choose treatment 1 relative to those who choose treatment 0. You are asked to evaluate the expected value of R_N. How should you respond? Explain.

Prelim Question, July 2018
Part II: 1 hour, 60 points

Do all problems, show all work, and define any symbols you use that are different from those in the problem statements.

Let the scalar variables Y and X be related by

$$Y = \beta X + U; \; E(UX) = 0,$$

where, $|X| \le C_X$, for some constant $C_X < \infty$. Assume that the distribution of U conditional on X has as many moments as you like and that the moments are all bounded by a finite constant C_U. Let $\{Y_i, X_i : \; i = 1,...,n\}$ be an independent random sample of (Y, X).

Problem 1 (13 points)

Let $\hat{b}_{OLS}$ be the ordinary least squares estimator of β. Find the variance of the asymptotic distribution of $n^{1/2}(\hat{b}_{OLS} - \beta)$. Denote this variance by σ^2_{OLS}.

Problem 2 (13 points)

Suppose you have the additional information that U is independent of X and $E(U) = E(X) = 0$. Let $\hat{b}_{GMM}$ be the GMM estimator of β based on the moment conditions $E(U) = 0$ and $E(UX) = 0$ with the asymptotically optimal weight matrix. What is the variance of the asymptotic distribution of $n^{1/2}(\hat{b}_{GMM} - \beta)$? Is it larger than, smaller than, or the same as σ^2_{OLS} under the assumptions of this problem?

Problem 3 (21 points)

As in Problem 2, suppose that U is independent of X and $E(U) = E(X) = 0$. Is there an estimator of β that is asymptotically more efficient than $\hat{b}_{OLS}$? In other words, is, is there an estimator $\hat{b}_{IND}$ such that the variance of the asymptotic distribution of $n^{1/2}(\hat{b}_{IND} - \beta)$ is less than σ^2_{OLS}? If yes, display such an estimator and show that the variance of its asymptotic distribution is less than σ^2_{OLS}. If no, prove it. The answer to this question may depend on the distribution of U. If it does, explain why.

Problem 4 (13 points)

The ordinary least squares estimator $\hat{b}_{OLS}$ is the best linear unbiased estimator (BLUE). If your answer to problem 3 is yes, explain why it does not conflict with the BLUE property of $\hat{b}_{OLS}$.

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Part III (60 points) - July, 2018

Question 1 60

Consider a model where $(Y(0), Y(1))$ denote potential outcomes and $D \in \{0,1\}$ is a treatment indicator. The observed outcome is $Y = DY(1) + (1-D)Y(0)$ and, for a scalar random variable $X \in \mathcal{X} \subseteq \mathbf{R}$ with $\mathcal{X}$ compact, it is assumed that

$$(Y(0), Y(1)) \perp\!\!\!\perp D \mid X \ . \tag{1}$$

Denote by $p(x) = P\{D = 1|X = x\}$ the propensity score and assume that $c \le p(x) \le 1-c$ for all $x \in \mathcal{X}$ and some $c > 0$. We observe a random sample of size n, $(Y_1, D_1, X_1), \ldots, (Y_n, D_n, X_n)$, and use this sample to compute the following weighted least squares estimators:

$$\begin{pmatrix} \hat{\beta}_0(x) \\ \hat{\beta}_1(x) \end{pmatrix} = \left(\sum_{i=1}^n k_i(x) Z_i(x) Z_i(x)' \right)^{-1} \sum_{i=1}^n k_i(x) Z_i(x) D_i Y_i \tag{2}$$

$$\begin{pmatrix} \hat{\gamma}_0(x) \\ \hat{\gamma}_1(x) \end{pmatrix} = \left(\sum_{i=1}^n k_i(x) Z_i(x) Z_i(x)' \right)^{-1} \sum_{i=1}^n k_i(x) Z_i(x)(1-D_i) Y_i \tag{3}$$

$$\begin{pmatrix} \hat{\pi}_0(x) \\ \hat{\pi}_1(x) \end{pmatrix} = \left(\sum_{i=1}^n k_i(x) Z_i(x) Z_i(x)' \right)^{-1} \sum_{i=1}^n k_i(x) Z_i(x) D_i \ , \tag{4}$$

where, for each $x \in \mathbf{R}$ and given $h \in \mathbf{R}$, $k_i(x) = k\left(\frac{X_i - x}{h}\right)$ is a second order kernel and

$$Z_i(x) = (1, X_i - x)' \ .$$

Assume throughout that the conditions for these estimators to be uniformly consistent (over $\mathcal{X}$) for the parameters they intend to estimate are satisfied in this case.

(a) (10 points) Show that the average treatment effect

$$\theta = E[Y(1) - Y(0)]$$

is identified under the above assumptions.

(b) (10 points) Consider estimating θ by taking the difference of the average of the outcomes of treated and control units. That is,

$$\hat{\theta}_n = \frac{1}{n_1} \sum_{i=1}^n D_i Y_i - \frac{1}{n - n_1} \sum_{i=1}^n (1 - D_i) Y_i$$

where $n_1 = \sum_{i=1}^n D_i$. Find the probability limit of $\hat{\theta}_n$ under (1) and argue whether $\hat{\theta}_n$ is consistent for θ or not .

(c) (10 points) Let $\mu_1(x) = E[Y(1)|X = x]$ and $\mu_0(x) = E[Y(0)|X = x]$. Show that $\mu_1(x)$ is identified by a ratio of two expectations. Conclude that the same is true for $\mu_0(x)$.

(d) (10 points) Now consider the estimators in (2)-(4). Explain what each of these estimators is estimating and describe the family of estimators they belong to.

(e) (10 points) Using the estimators in (2)-(4), consider inputing values for $Y(1)$ and $Y(0)$ as follows:

$$\hat{Y}_i(1) = D_i Y_i + (1 - D_i)\frac{\hat{\beta}_0(X_i)}{\hat{\pi}_0(X_i)}$$
$$\hat{Y}_i(0) = (1 - D_i) Y_i + D_i \frac{\hat{\gamma}_0(X_i)}{\hat{\pi}_0(X_i)} ,$$

that is, you set $\hat{Y}_i(1) = Y_i$ if $D_i = 1$ for the i^{th} unit and $\hat{Y}_i(1) = \frac{\hat{\beta}_0(X_i)}{\hat{\pi}_0(X_i)}$ otherwise (similarly for $\hat{Y}_i(0)$). Find the probability limit of

$$\tilde{\theta}_n = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i(1) - \hat{Y}_i(0))$$

under (1) and argue whether $\tilde{\theta}_n$ is consistent for θ or not.

(f) (10 points) Finally, suppose $p(X)$ were known and consider estimating θ by GMM using the moment condition $E[\Psi(Y, D, p(X), \theta)] = 0$ where

$$\Psi(Y, D, p(X), \theta) = \frac{[D - p(X)]Y - \theta p(X)(1 - p(X))}{p(X)(1 - p(X))} .$$

Denote this estimator by $\bar{\theta}_n$ and argue it is unbiased and consistent for θ under (1).