2018 Microeconomics Prelim

This exam is divided into three question: one each for 410-1, 410-2 and 410-3.

Please note: each question has a maximum of 30 points possible and
has the SAME WEIGHT in the exam.

Each question is divided into parts and sub-parts. Partial credit is awarded as indicated.

You should attempt to solve each question.

410-1 Question

A firm has $\bar{x}$ units of an input to allocate among n plants, with $\infty > n > 2$. With probability p_i, $i = 1, ...n$, where $\sum p_i = 1$ and $p_i > 0$ for all i, only plant i survives and if the firm allocated x_i units of the good to plant i then her profit (and revenue since there are no costs) will be $u(x_i)$. Assume only that u is **weakly increasing** and as differentiable as you like. She (the firm) chooses how to allocate the goods she has among the plants to maximize ex ante expected profit.

Let $W(p, \bar{x})$ be the maximum ex ante expected profit she can get with endowment $\bar{x}$ and probabilities p. Let $X(p, \bar{x})$ be the allocation of $\bar{x}$ to the n plants that achieves the maximum.

Note that p is a probability vector, not any possible vector in R^n_+, so all these functions are defined only for the domain of probability vectors.

1. Write the optimization problems that W solves. Write the FOC.

2. Is $X(p, \bar{x})$ necessarily convex valued? That is, if x^* and x^{**} are both in $X(p, \bar{x})$, then are convex combinations of them in $X(p, \bar{x})$? If yes, prove this, if not give a counter counter example and specify a sufficient condition on u under which it is. (Whether or not an additional condition is needed, we will *not* assume it henceforth.)

3. Is $W(p, \bar{x})$ convex/concave/quasiconvex/quasiconcave in p? Prove only those properties that are true without making additional assumptions on u than those that are specified in the question. You do not need to provide a counterexample for properties that you do not claim are true.

4. Is $W(p, \bar{x})$ convex/concave/quasiconvex,/quasiconcave in $\bar{x}$? Prove only those properties that are true without making additional assumptions on u than those that are specified in the question. You do not need to provide a counterexample for properties that you do not claim are true.

5. Order p so that $p_1 < \cdots < p_n$. What can you say about the relationship between X_i and X_j for $i < j$? Do not make additional assumptions on u than were given.

6. Assume there are two firms with two different **strictly concave** production functions, u and v, that solve the same problem, i.e., given the same $\bar{x}$ and the same p, and where $-u''(x)/u'(x) < -v''(x)/v'(x)$ for all x. Let $p_1 < \cdots < p_n$. What can you say about the relationship between the optimal solutions for plant 1 for the two firms? **Prove** your answer.

410-2 Question

Consider an economy with I agents in which each agent i' utility depends upon "money" $m \in \mathbb{R}$ and a "policy" variable $x \in \mathbb{R}$:

$$u_i(m_i, x) = m_i - (b_i - x)^2,$$

where $b_i \in \mathbb{R}$.

The level of the policy variable x is determined by "lobbying resources" (think of this as effort, ability, time...) expended by agents. Given the vector of lobbying choices $x_1, \ldots, x_I$, the policy variable equals $x = \sum_i x_i$.

Agents can trade money and lobbying in competitive markets. Each agent is endowed with 0 units of money and 1 unit of lobbying resources. The price of money is normalized to 1, and the price of lobbying is denoted by p. There is *free disposal of lobbying resources*: thus, feasibility and market clearing are defined with a *weak inequality* for lobbying (but equality for money).

Both the endowment and the price of lobbying refer to the *absolute value* of lobbying resources. For instance, an agent who chooses either $x_i = \frac{1}{2}$ or $x_i = -\frac{1}{2}$ "spends" $\frac{1}{2}$ unit of lobbying. Also, $x_i = -2$ has a cost of $2p$ for agent i, as does $x_i = 2$. (One way to think of this is to imagine that the agent chooses the *amount*, or size, and *direction*, or sign, of her lobbying effort, and the price paid depends upon the amount only.) If an agent exerts lobbying effort x_i with $|x_i| > 1$, the interpretation is that she is buying lobbying resources from some other agent.

A *competitive lobbying equilibrium* is a price p^* for lobbying, and choices (m_i^*, x_i^*) for each agent $i \in I$, such that:

(i) for every $i \in I$, (m_i^*, x_i^*) maximizes i's utility $u_i(m_i, x_i + \sum_{j \neq i} x_j^*)$ over the budget constraint $\{(m_i, x_i) : m_i + p^*|x_i| \leq p^*\}$, given the equilibrium price p^* and lobbying choices $(x_j^*)_{j \neq i}$ of the other agents; and

(ii) markets clear: $\sum_i m_i^* = 0$ and $\sum_i |x_i^*| \leq I$.

The first four parts help you characterize agent i's optimal policy. One must distinguish cases. It is useful to define $x_{-i} \equiv \sum_{j \neq i} x_j$.

(a) 2 points. Argue that agent i's problem has a unique solution, and that it is enough to determine the optimal choice of x_i.

(b) 4 points. Assume that the optimal x_i is non-negative; write down the required first-order condition and derive an inequality, involving p, b_i and x_{-i}, that must hold for the optimal x_i to indeed be non-negative.

(c) 4 points. Now repeat the analysis for the case of a non-positive optimal x_i.

(d) 3 points. Argue that, if neither of the conditions in (c) and (d) holds, then the optimal x_i must be zero.

Now turn to equilibrium analysis. First, a symmetric case: assume that $b_i = b > 0$ for all i.

(e) 7 points Assume that $b \leq I$. Construct a symmetric equilibrium in which every agent's choice of lobbying x_i is the same, and compute the equilibrium price and policy outcome $x = \sum_i x_i$. [Note: if $b < I$, free disposal of lobbying is required (and allowed).]

Now assume that $I = 2J$ for some $J \geq 1$, and that agents $i = 1, \ldots, J$ are characterized by $b_i = b > 0$, whereeas $b_i = 0$ for agents $i = J+1, \ldots, I = 2J$.

(f) 10 points Construct an equilibrium in which every agent $i = 1, \ldots, J$ choose $x_i = \beta > 0$, every agent $i = J+1, \ldots, 2J$ chooses $x_i = -\gamma$ (with $\gamma > 0$), and the market for lobbying clears with equality. Compute the equilibrium price, lobbying choices, and policy outcome.

410-3 Question

A worker has one of three types: $i = 1$ with prob. 20%, 2 with prob. 40%, or 3 with prob. 40%. Only the worker knows his type. An employer considers hiring this worker. Type i has value $v = i$ to the employer. The worker also has an option of starting his own business, in which case the worker of type i will make $i - 1$.

Consider the model (game) in which Nature first chooses the worker's type. Next, the employer offers the worker a wage w at which she will be willing to hire the worker. Finally the worker decides whether to accept the offer or to start his own business. The payoff of the employer is 0 if the worker rejects the offer, and $v - w$ if the worker accepts the offer. The employer is an expected-payoff maximizer. The payoff of the worker is w if he accepts the offer, and $i - 1$ if he rejects the offer (and starts his own business).

(a) (3 points) Draw the extensive form of this game.

(b) (3 points) What values of wage w guarantee that type 1 will accept the offer? What values guarantee that type 2 will, and what values guarantee that type 3 will?

(c) (i) (3 points) Find a weak PBE of this game. (ii) (3 points) Is this equilibrium also sequential? [Part (ii) can be answered in one short sentence.]

Suppose now that types 2 and 3 of the worker can take a training, which has NO effect on their value, but can be used for signaling purposes. The cost of taking the training is 1/2 for each of these types. Type 1 cannot take the training, e.g., this type would not be able to pass the final exams. The training takes place before the employer makes an offer to the worker, and the employer knows whether the worker took the training. So, the employer can now offer different wages: w_N - if the worker did not take the training, and w_Y - if the worker took the training.

(d) (9 points) Show that in any weak PBE neither type 2 nor type 3 take the training.

Suppose finally that there are two identical employers competing for the service of the worker. The employers make their wage offers simultaneously, after observing if the worker took the training. [You may be able to answer parts (e) and (f), even when you did not answer parts (a)-(d).]

(e) (3 points) If the employers knew that both type 2 and type 3 took the training, what would be their offer w_Y in a Nash equilibrium?

(f) (6 points) Show that there is a weak PBE in which both types 2 and 3 take the training? [You must specify w_Y and w_N, the employers' beliefs contingent on the worker taking the training, the beliefs contingent on not taking the training, and show that the strategies are sequentially rational.]