Econometrics Preliminary Exam
July, 2019

Read the instructions below before starting the exam:

- The preliminary exam consists of **three parts** (one for each quarter of Econ 480).
- You have 180 minutes (1 hour for each part) to solve this exam.
- This is a closed book and notes exam.
- You can use one page (both sides) of hand-written or typed notes.
- You may use a calculator if needed
- Please keep your written answers brief; be clear and to the point.
- We recommend that you circle or otherwise indicate your final answers.
- There is no choice, so remember to answer all questions.
- Good luck!

PLEASE WRITE EACH PART OF THE EXAM IN
A SEPARATE BLUE BOOK.

2019 Econometrics Prelim, Part I (one hour, 60 points)

1. Suppose that a product is offered at price p per unit. Suppose that each person j in population J chooses between purchase of zero or one unit of the product. Let

x_j be a binary covariate for person j, taking the value zero or one.

e_j be the resource endowment available to person j. It is the case that $e_j > 0$.

v_j be the valuation person j places on the product. That is, v_j is the highest price the person would be willing to pay for one unit. Suppose that $0 \leq v_j \leq e_j$.

$z_j = 1$ if person j chooses to purchase one unit at price p, and $z_j = 0$ if j does not purchase the product.

Let P(v, x, e, z) denote the population distribution of the variables (v, x, e, z). One observes (x_j, e_j, z_j) for each person $j \in J$. However, one does not observe v_j.

A. (15 points) What can one conclude about E(v)?

B. (5 points) A researcher assumes that $E(v|x = 1) = E(v|x = 0)$. What can he or she conclude about E(v)?

2. Let J be a population, each member j of which receives one of the two treatments a or b. Let $y_j(t)$ be a bounded outcome under treatment t, with $y_j(t)$ taking values in the unit interval [0, 1]. Let P(y, z) be the observed population distribution of realized outcomes and treatments.

Suppose it is found that $y \perp z$; that is, y is statistically independent of z. In the absence of other information, some researchers make assertions about the identification region $H\{E[y(b)] - E[y(a)]\}$ for the average treatment effect. Evaluate these assertions:

Assertion A (8 points): $y \perp z \;\Rightarrow\; H\{E[y(b)] - E[y(a)]\} = \{0\}$.

Assertion B (8 points): $y \perp z \;\Rightarrow\; 0 \in H\{E[y(b)] - E[y(a)]\}$.

Assertion C (9 points): $y \perp z \;\Rightarrow\;$ 0 is the center of $H\{E[y(b)] - E[y(a)]\}$.

In each case, explain your answer.

3. (15 points) Let J be a population, each member j of which may receive a real-valued treatment t, taking values in the unit interval [0, 1]. Let $y_j(t)$ be a bounded outcome under treatment t, with $y_j(t)$ taking values in the unit interval [0, 1]. Let F denote the space of concave functions mapping $[0, 1] \to [0, 1]$. Let it be known that $y_j(t) = f(t)$ for some $f(\cdot) \in F$. Thus, it is known that all persons have the same concave treatment response function, but the specific form of this function is not known.

A planner can assign each agent any feasible action. Thus, the planner can choose any element of the Cartesian Product set $[0, 1]^{|J|}$. Let $(w_j, j \in J)$ be any treatment allocation. Suppose that the planner wants to choose an allocation to maximize the mean outcome. Thus, the planner wants to choose $(w_j, j \in J)$ to maximize $\int f(w_j)dP(j)$. Suppose that the planner considers two treatment allocations, being

Allocation I: a fractional allocation assigning 1/3 of the members of the population to each of the three treatment values w = 0, w = ½, and w = 1.

Allocation II: the singleton allocation assigning all members of the population to treatment value w = ½.

What, if anything, can you say about which of these two allocations the planner should prefer? Explain.

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Part II (60 points) – July, 2019

Do all problems, show all work, and define any symbols you use that are different from those in the problem statements. Explain why each answer you give is correct. You will not receive credit for an answer, even if it is correct, without explaining why it is correct.

1. (20 points)

Let Y be a continuously distributed random variable with an "ordinary" probability density function that is non-zero everywhere on the interior of its support. Let $\{Y_i : i = 1,...,n\}$ be an independent random sample of Y. Let $p \geq 1$ be an integer. Define $\hat{\theta}$ and $\tilde{\theta}$ by

$$\hat{\theta} = \arg\min_t \sum_{i=1}^{n} | Y_i - t |^p$$

$$\tilde{\theta} = \arg\min_t \sum_{i=1}^{n} (Y_i - t)^p .$$

Assume that θ is contained in a compact interval and that this interval contains the probability limits of $\hat{\theta}$ and $\tilde{\theta}$ whenever they exist. Let $n \to \infty$. Define a central moment of the distribution of Y as a moment of Y about its mean if a finite mean exists.

a. If $p = 1$, under what conditions, if any, does $\hat{\theta}$ converge in probability to a quantile or central moment of the distribution of Y? What is that quantile or moment?

b. If p is an even integer, under what conditions, if any, does $\hat{\theta}$ converge in probability to a central moment of the distribution of Y? What is that moment?

c. If p is an odd integer, under what conditions, if any, does $\tilde{\theta}$ converge in probability to a central moment of the distribution of Y? What is that moment?

2. (20 points)

Suppose you want to test the hypothesis H_0 that a certain random variable X has the exponential distribution with density

(1) $f(x,t) = t\exp(-xt);\ t \geq 0$.

If H_0 is true, the information equality gives

(2) $$E\frac{\partial^2 \log f(X,t)}{\partial t^2} + E\left(\frac{\partial \log f(X,t)}{\partial t}\right)^2 = 0,$$

where $f(x,t)$ is the function (1).

a. Let $\{X_i : i = 1,...,n\}$ be an independent random sample of X. Use (2) to derive a statistic for testing the hypothesis that the density of X is (1).

b. Is there a density of X that is different from (1) but for which (2) holds under the erroneous assumption that the density is (1)? If yes, give an example of such a density. If no, prove it.

3. (20 points)

Let

$$Y = \beta_0 X + U,$$

where Y, X, and U are random variables, U is unobserved, and X may be correlated with U. The random variable Z is called an instrument for X if $E(ZU) = 0$ and $E(ZX) \neq 0$. Let $\{Y_i, X_i, Z_i : i = 1,...,n\}$ be an independent random sample of (Y, X, Z). The instrumental variables (IV) estimate of β_0 is

$$\hat{\beta}_{IV} = \frac{\sum_{i=1}^{n} Y_i Z_i}{\sum_{i=1}^{n} X_i Z_i}.$$

Suppose that $E(X) = E(Z) = E(U) = 0$ and that (X, Z, U, XZ, XU, ZU) have as many finite moments as you like.

a. If $E(XZ) = r$ for some constant $r \neq 0$. Does $\hat{\beta}_{IV} \to^p \beta_0$ as $n \to \infty$? If yes, prove it. If no, give a counterexample.

b. Z is called a weak instrument if $E(XZ) = n^{-1/2} r$ for some constant $r \neq 0$. Does $\hat{\beta}_{IV} \to^p \beta_0$ as $n \to \infty$ if Z is a weak instrument? If yes, prove it. If no, give a counterexample.

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Part III (60 points) - July, 2019

Question 1 60

Let (Y, X) be random variables taking values in $\mathbf{R}$ and suppose we are interested in estimating the conditional expectation of Y given X, i.e. $m(x) = E[Y|X = x]$. To do this, we use a random sample of size n from the distribution of (Y, X) and focus attention to estimators of the following form,

$$\hat{m}_n(x) = \sum_{i=1}^{n} \omega_i(x) Y_i \tag{1}$$

where $\{\omega_i(x) : 1 \leq i \leq n\}$ are weights that may depend on $(X_1, \ldots, X_n)$ but not on $(Y_1, \ldots, Y_n)$. Suppose (Y, X) have finite and positive second moments.

(a) Consider running a linear regression of Y on $(1, X)$ and define $\hat{m}_n^{LS}(x) = \hat{\beta}_{n,0} + x\hat{\beta}_{n,1}$ where $(\hat{\beta}_{n,0}, \hat{\beta}_{n,1})$ are the least squares estimators of the intercept β_0 and slope parameter β_1.

  i. (10 points) Show that $m_n^{LS}(x)$ takes the form in (1) with $\omega_i^{LS}(x) = \left(\frac{1}{n} + (x - \bar{X}_n)\frac{(X_i - \bar{X}_n)}{\sum_{i=1}^{n}(X_i - \bar{X}_n)^2}\right)$.

  ii. (5 points) What is the interpretation of $\hat{m}_n^{LS}(x)$ in this context?

  iii. (5 points) Are $(\hat{\beta}_{n,0}, \hat{\beta}_{n,1})$ unbiased for (β_0, β_1)?

(b) Consider the Nadaraya-Watson estimator of $m(x)$ and denote it by $\hat{m}_n^{NW}(x)$.

  i. (5 points) Write down the expression for $\hat{m}_n^{NW}(x)$ clearly explaining all the objects and notation that enter your expression.

  ii. (5 points) Show that $\hat{m}_n^{NW}(x)$ takes the form in (1) and provide a closed form expression for the weights, i.e., $\omega_i^{NW}(x)$.

(c) Consider the Local Linear (LL) estimator of $m(x)$ and denote it by $\hat{m}_n^{LL}(x)$.

  i. (5 points) Write down the expression for $\hat{m}_n^{LL}(x)$ clearly explaining all the objects and notation that enter your expression.

  ii. (5 points) Show that $\hat{m}_n^{LL}(x)$ takes the form in (1) with

$$\omega_i^{LL}(x) = e' \left(\sum_{i=1}^{n} k_i(x) Z_i(x) Z_i(x)'\right)^{-1} k_i(x) Z_i(x) \ ,$$

  where $k_i(x) = k((X_i - x)/h)$ is a second order kernel, $Z_i(x) = (1, (X_i - x))$, and $e' = (1, 0)$.

(d) Suppose it is known that the model is "approximately linear", in the sense that

$$m(x) = \theta x + r(x) \quad \text{with} \quad |r(x) - r(x')| \leq M|x - x'| \ , \quad \theta \in \mathbf{R}, \text{ and } M \in (0, 1) \ .$$

  i. (5 points) Show that for any estimator taking the form in (1),

$$E[\hat{m}_n(x)|X_1, \ldots, X_n] = m(x) \sum_{i=1}^{n} \omega_i(x) + \sum_{i=1}^{n} \omega_i(x)(X_i - x)\theta + R_n \ ,$$

  where R_n is a reminder term. Provide an expression for R_n.

ii. (5 points) Show that whenever the weights satisfy

$$\sum_{i=1}^{n} \omega_i(x) = 1 \text{ and } \sum_{i=1}^{n} \omega_i(x)(X_i - x) = 0 \ , \tag{2}$$

the bias $|E[\hat{m}_n(x)|X_1, \ldots, X_n] - m(x)|$ is bounded by an expression that does not depend on θ. Provide an expression for this upper bound.

iii. (10 points) Consider the three estimators you previously discussed: $\hat{m}_n^{LS}(x)$, $\hat{m}_n^{NW}(x)$, and $\hat{m}_n^{LL}(x)$. Show that two of these estimators satisfy the requirement in (2) and provide an interpretation of why this is the case.