Macroeconomics Prelim

Friday, July 19, 2019

Please answer all four questions. The number of points for each question is indicated in parenthesis. If you find a question to be ambiguous, say why, sharpen up the question, and proceed. Please answer each question in a separate blue book. Good Luck!

Question 1 - Equilibrium as Fixed Point of Best Response Function (*25 pts*)

Consider the following one-period economy. There is a large number of identical households and each solves the following problem:

$$\max_{c,l\geq 0} u\left(c,l\right) = \log c - l,$$

subject to its budget constraint:

$$c \leq wl + rk,$$

where k denotes the level of capital owned by the household and supplied inelastically to the capital rental market. Also, c and l denote consumption and labor of the typical household. The household takes the market wage, w, and rental rate of capital, r, as given and beyond its control.

A representative competitive firm operates the following production function:

$$Y = K^{\alpha} L^{1-\alpha},\ 0 < \alpha < 1/2,$$

where Y, K and L denote the per capita amount of output, capital and labor, respectively. The firm seeks to maximize profits by choosing L, K to maximize

$$Y - wL - rK,$$

subject to its production function. A *competitive equilibrium* is a w, r, l, L, k, K, c, Y such that: (i) given the prices the household and firm problems are solved, (ii) labor, capital and goods markets clear: $l = L, k = K$ and $c = Y$.

(a) Show that there is a unique equilibrium and display explicit formulas relating the equilibrium values of L, r, w, to the model parameters. (*2.5 points*)

(b) The concept of competitive equilibrium is silent about how people come to know the equilibrium prices, r and w. Since agents make their decisions simultaneously without communicating with each other, they must somehow form a belief about r and w on their own. One approach is to suppose that people learn from past observations, but this is ruled out here because there is only one period. Another approach supposes they arrive at a belief about r and w by privately working out the implications of common knowledge about rationality and the structure of each household's problem. This approach is implemented by transforming the market economy into a game between a continuum of identical agents. In this game, the individual household's 'strategy' is l and their conjecture about the per capita choice by other households is L (to simplify things, you may assume that when households form a belief, L, about what other households do, they suppose that each individual household does L). (*11.25 points*) (b.1) Show that, conditional on a specific belief, L, the typical household can deduce what the competitive wage rate and rental rate of capital will be. (*1.25 points*)

(b.2) Conditional on the r and w implied by the belief, $L \in [0, \infty)$, the household can determine the level of employment, l, that maximizes its own payoff, subject to its constraints. Denote the household's best response to L by $l = F(L)$. (Model parameters like K have been suppressed to avoid cluttering the notation.) Identify a simple function, $G(l, L) = 0$ for which F is the solution to the functional equation, $G(F(L), L) = 0$ for all $L \geq 0$. Provide the analytic representation for F. (*5 points*)

(b.3) What is the economic intuition behind the 'strategic substitute' property of the impact of L on l, $F' < 0$? (*2.5 points*)

(b.4) Show explicitly that there is a unique Nash equilibrium, $L = L^*$, $L^* = F(L^*)$, which coincides with the competitive equilibrium. (*2.5 points*)

(c) If the competitive equilibrium can be expressed as a Nash equilibrium of a game (as in the above example), in principle the problem of how the individual household forms its belief about r, w is solved: (i) compute the Nash equilibrium L^*, and (ii) solve for $r(L^*)$ and $w(L^*)$. (*11.25 points*)

(c.1) Provide a reason why the Nash approach is appealing. Give some reasons why it may not be appealing. (*1.25 points*)

(c.2) An alternative approach to thinking about how people might come up with a belief about r and w is *rationalizability*: people will only adopt strategies, l, based on beliefs about others' strategies, L, that are *justified*. A belief about another's strategy is justified if the other's strategy can itself be rationalized by some justifiable belief about everyone else's strategy, and so on. The set of rationalizable strategies is usually obtained by a procedure which deletes candidate beliefs iteratively. For example, consider an individual household that begins by contemplating the entire range of logically possible beliefs, $L \in [0, \infty)$. Explain why, under common knowledge, the individual chooses to delete $L > 1$ on the grounds that such beliefs are not justified. (*2.5 points*)

(c.3) Suppose the individual only contemplates beliefs on the unit interval $0 \leq L \leq 1$. Will the individual choose to delete additional beliefs as unjustified, given that $L > 1$ has been ruled out? Make your argument using a graph with l on the vertical axis and L on the horizontal. Include a graph of the best response function as well as the 45 degree line.(*2.5 points*) (c.4) Argue that as you consider successive deletions of beliefs, as in c.2 and c.3, the set of justifiable L's shrinks to a set with only one element, the Nash equilibrium (i.e., L^*). This result gives us a basis for predicting that under common knowledge, households will choose the Nash belief. Similarly the assumption in a competitive equilibrium, that households know w and r, can be motivated by the assumption of common knowledge of rationality and the structure of the economy. (*2.5 points*)

(c.5) Show that the assumption, $\alpha < 1/2$, plays a crucial role in the result in c.4. In particular, for $1 > \alpha > 1/2$ common knowledge of rationality does not provide a basis for the assumption that households know the values of r and w. (*2.5 points*)

Question 2 - Labor Supply in Business Cycle Models (*25 pts*)

Consider the social planner's problem for a real business cycle model. The household makes consumption (C) and leisure ($1-N$), where N is hours worked decisions to maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t). \tag{1}$$

Specific functional forms will be given below.

Output is produced using capital K and labor N

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha}. \tag{2}$$

Z_{t} is a TFP (total factor productivity shock) and is governed by a discrete state Markov chain. Capital evolves according to:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{3}$$

Assume there is full depreciation .

$$\delta = 1.$$

There is no trend growth and

$$Y_t = C_t + I_t. \tag{4}$$

First suppose that the utility function u is as follows:

$$ln\left(C_{\mathrm{t}} - \frac{N_t^2}{2}\right) \tag{5}$$

(a) Write down the recursive formulation of planner's problem and derive the first order conditions. What is the relationship between the Frisch and regular elasticity of labor supply when u is given by (5)? (*5 pts*)

(b) Using guess and verify, find the policy functions for investment, consumption and hours worked (*Hint:* first consider the equilibrium condition for hours worked and guess that investment is a constant share of output). (*7.5 pts*)

Now suppose the utility function is given by

$$ln(C_t) - \frac{N_t^2}{2}$$

(c) Repeat parts (a) and (b) using these new preferences. (*5 pts*)

(d) Compare the business cycle properties implied by these two models and explain how and why a TFP shock might affect output, consumption, investment and hours worked. Some RBC modelers prefer preferences used in parts (a/b) to those in part (c), why might this be the case? Hint: your answer should involve the different income effect of a technology shock implied by the two different specifications. (*7.5 pts*)

Question 3 - Expectational Errors in the New Keynesian Model (*25 pts*)

Suppose that we have the standard intertemporal Euler equation and New Keynesian Phillips curve[1], and there is perfect foresight, except that an *expectational error* η_t in households' expectations at date t of period $t+1$ inflation can appear in the Euler equation (but not in the NKPC, where firms' expectations are correct):

$$\hat{y}_t = \hat{y}_{t+1} - \sigma^{-1}(i_t - \pi_{t+1} - \eta_t - \rho) \tag{6}$$

$$\pi_t = \beta\pi_{t+1} + \kappa\hat{y}_t \tag{7}$$

As usual, monetary policy optimizes a quadratic loss function of the form

$$\sum_{t=0}^{\infty} \beta^t(\hat{y}_t^2 + \lambda_\pi \pi_t^2) \tag{8}$$

Suppose that $\eta_0 < 0$ and $\eta_t = 0$ for all $t > 0$, and that this is known by the policymaker.

1. Suppose i_t is unrestricted. What is the path for i_t at the optimum?[2] (*6 pts*)

2. Suppose that there is a zero lower bound (ZLB) constraint: $i_t \geq 0$. For what values of η_0 is this binding for optimal policy? (*6 pts*)

3. If the policymaker optimizes under *discretion*, i.e. it reoptimizes (8) at each date t to maximize utility $\sum_{t'=t}^{\infty} \beta^{t'-t}(\hat{y}_{t'}^2 + \lambda_\pi \pi_{t'}^2)$ from t onward, what is the equilibrium under optimal policy when the ZLB binds? (*6 pts*)

4. Suppose that the policymaker optimally commits at date 0 to policy at dates 0 and 1, but that it cannot commit to policy at later dates and optimizes under discretion from date 2 onward. Also, suppose that the ZLB binds *only* at date 0, and that $\kappa = 0$, so that $\pi_t = 0$ for all t. Solve for equilibrium and give intuition for the path of $\hat{y}_t$. How does being able to commit to policy at date 1 help? (*7 pts*)

[1] We write $\hat{y}_t$ rather than $\tilde{y}_t$ since there are no productivity or other shocks that could lead to variation in the natural level of output.

[2] Do not worry about uniqueness or implementation of equilibrium in any of these questions; just find equilibrium sequences that optimize (8) subject to (6), (7), and any other constraints on central bank behavior that we impose.

Question 4 - Housing Services in a Permanent Income Model (*25 pts*)

Consider a consumer with preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where c_t are non-durable goods and h_t are housing services. Housing services are produced, one for one, from units of housing. Consider a consumer who is receiving a constant income stream $y_t = y$ and faces no borrowing constraints.

1. *(4 pts)* Write the per-period budget constraint assuming the consumer can buy and sell housing at the price P_t each period, with no transaction costs and no adjustment costs. Assume housing units depreciate at the rate δ. Assume the interest rate is constant at r. Use $(1+r)\, a_{t-1}$ to denote the consumer net asset position (excluding housing wealth) at the beginning of the period t.

2. *(4 pts)* Adding up the per-period budget constraints and using an appropriate no-Ponzi condition, write an intertemporal budget constraint for the sequences $\{c_t, h_t\}$.

3. *(4 pts)* Show that housing costs in the intertemporal budget constraint can be re-arranged to appear as
$$\sum_{t}^{\infty} \left(\frac{1}{1+r}\right)^t \left(P_t - \frac{1-\delta}{1+r} P_{t+1}\right) h_t.$$
(If you already did this in (ii) that's fine.) Provide an interpretation for the expression
$$P_t - \frac{1-\delta}{1+r} P_{t+1}.$$

4. *(4 pts)* Write down the consumer optimization problem and derive first order conditions for c_t and h_t, assuming preferences take the form
$$u(c_t, h_t) = \alpha \ln c_t + (1-\alpha) \ln h_t.$$

5. *(5 pts)* Assume the consumer starts at $t = 0$ with zero housing and non-housing wealth, $h_{-1} = a_{-1} = 0$. Assume that $(1+r)\beta = 1$ and assume house prices grow at a constant rate: $P_{t+1} = (1+g) P_t$. Show that the consumer chooses constant levels for c_t and $P_t h_t$ and characterize these levels as much as you can.

6. *(4 pts)* Consider the comparative statics of your results in (v), when you increase the rate g at which consumers expect house prices to grow (keeping P_0 constant). What happens to the value of housing purchases $P_0 h_0$, what happens to consumption c_0? Why?