2019 Microeconomics Prelim

This exam is divided into three question: one each for 410-1, 410-2 and 410-3.

Please note: each question has a maximum of 30 points possible and
has the SAME WEIGHT in the exam.

Each question is divided into parts and sub-parts. Partial credit is awarded as indicated.

You should attempt to solve each question.

410-1 Question

Consider $f : \mathbf{R}^n \times \mathbf{R} \to \mathbf{R}$, where $n \geq 2$ and we denote the variables as follows: $f(x,t)$, $x \in \mathbf{R}^n$, $t \in \mathbf{R}$.

Let $X^*(t) = \arg\max_x f(x,t)$. Assume throughout this set is non-empty.

Notation: I write $x \geq x'$ for $x_i \geq x'_i$ for $i = 1, ..., n$.

In class we studied an MCS result providing assumptions on f under which we could conclude that when t increases X^* increases in the strong set order. We will consider a related result here. Below is a new definition and a reminder of some familiar definitions.

Definition 1 *f satisfies increasing differences in x and t if for all $x' \geq x$ and $t' \geq t$, $f(x',t') - f(x,t') \geq f(x',t) - f(x,t)$.*

Definition 2 *$X^*(t)$ is increasing in the strong set order if for $t' > t, x \in X^*(t)$ and $x' \in X^*(t')$ we have that the component-wise minimum of x and x', $(\min\{x_i, x'_i\})_{i=1}^n$, is in $X^*(t)$ and the component-wise max is in $X^*(t')$.*

Definition 3 *$X^*(t)$ is nowhere decreasing in t if $t' > t, x' \in X^*(t'), x \in X^*(t)$ and $x \geq x'$ imply $x \in X^*(t')$ and $x' \in X^*(t)$.*

(a) **4 points.** If $X^*(\cdot)$ is increasing in the strong set order is it nowhere decreasing? Is the converse true? Provide a *brief* argument if yes and a counterexample if no.

Claim 4 *If f satisfies increasing differences in x and t then $X^*(\cdot)$ is nowhere decreasing.*

(b) **2 points.** How does this claim differ in terms of assumptions and conclusion from the multi-dimensional MCS result presented in class?

(c) **5.5 points.** Prove this claim.

(d) **5.5 points.** Use this claim (and you are not to prove this using any other methods) to prove that, for a profit-maximizing price-taking firm with production function $g : \mathbf{R}^n_+ \to \mathbf{R}_+$ which has a unique optimal solution at any price vector, when the price of an input increases it cannot be that the demand of all outputs strictly increase. Provide a precise and complete argument.

(e) **3 points.** What assumption could you add to part (d) to conclude that demand of *all* inputs weakly decrease? Explain your answer briefly but precisely.

410-2 Question

Consider an economy under uncertainty with I agents, a single consumption good (chocolate), and state space $S = \{1,2\}^I$. The contingent consumption space is $X_i = \mathbb{R}^S_{++}$ for each agent. Furthermore, agents have identical EU preferences over contingent consumption: there is a common probability distribution $\pi \in \Delta(S)$ such that, if i eats $x_{si} \in \mathbb{R}_{++}$ units of chocolate in state $s \in S$, her expected utility is

$$u(x_i) = \sum_{s \in S} \pi_s \ln x_{si}. \tag{1}$$

The i-th coordinate of S determines agent i's endowment:

$$\forall s = (s_1, \ldots, s_I) \in S = \{1,2\}^I, \qquad \omega_{si} = s_i. \tag{2}$$

To sum up, this is a standard economy under uncertainty à la Arrow-Debreu, in which the i-th coordinate of the state space represents agent i's *individual risk* (about her endowment).

(a) 5 points. Compute the unique Arrow-Debreu equilibrium price vector $p^* \in \mathbb{R}^S_{++}$ and contingent consumption $x^*_i \in \mathbb{R}^S_{++}$. NOTE: normalize the price vector p so that $\sum_{i \in I} \sum_{s \in S} p_s \omega_{si} = 1$. This gives a simple expression for prices and demands.

(b) 3 points. Show that, in (a), agent i's contingent consumption in a given state s can be written as a constant fraction of the realized *total* endowment $\sum_{j \in I} \omega_{sj}$, where the constant equals i's *expected* share of the total endowment, i.e., $\sum_{t \in S} \pi_t \frac{\omega_{ti}}{\sum_{j \in I} \omega_{tj}}$.

For **all remaining points of this question**, assume that the common probability π satisfies the following *exchangeability* property: for every permutation $i_1, \ldots, i_I$ of $1, \ldots, I$,

$$\pi(s_1, \ldots, s_I) = \pi(s_{i_1}, \ldots, s_{i_I}). \tag{3}$$

[A special case is that of IID realizations for each individual, but Eq. (3) allows for correlation as well.]

(c) 6 points. Show that each agent's expected share of the total endowment is $\frac{1}{I}$. [HINT: compute each agent's expected share conditional on there being exactly n high endowment realizations. To do so, what is the probability that agent i has a high endowment realization, conditional on n agents having high realizations?]

(d) 2 points. Argue that, if the same number of agents has a high endowment in two states $s, t \in S$, then $\pi_s = \pi_t$, $p^*_s = p^*_t$, and $x^*_{si} = x^*_{ti}$ for all i.

Recall that an economy has *no aggregate uncertainty* if the total endowment is state-independent (i.e., it is the same in all states). Also recall that, with a common prior, in an economy with no aggregate uncertainy, in equilibrium agents fully insure—that is, their consumption is also state-independent. In general, the economy considered here does feature aggregate uncertainty. However:

(e) 4 points. Using (d), show that a "conditional" version of this full-insurance property holds in this economy.

410-3 Question

A worker of privately known ability $\theta \in \{H, L\}$ decides whether to take a two-year MBA program. The MBA degree is costly to obtain, but does not affect worker's on-the-job productivity. The cost of obtaining the MBA degree is smaller for a high-ability worker. This enables such workers to signal their type to potential employers, who are seeking to hire only high-ability workers.

Consider the following game. First, nature determines the worker's type $\theta \in \{H, L\}$ with $\Pr(H) = p_0 \in (0, 1)$. After observing own type, the worker chooses $e \in \{0, 2\}$, that is, whether to take the program ($e = 2$), or not ($e = 0$). Then, the firm, which observes the worker's choice of e but not the worker's type θ, chooses $a \in \{Y, N\}$, that is, whether to hire the worker ($a = Y$), or not ($a = N$).

The payoffs are as follows. If the firm hires the worker, the firm pays an exogenous wage w to the worker. If the employee is of high ability, then the firm also obtains a revenue of 1; otherwise the revenue is zero. The worker's payoff is equal to wage w minus the cost of education e/θ. If the firm does not hire the worker, then the firm's payoff is zero, but the worker incurs the cost of education. All players are risk-neutral.

Assume that $2/H < w < 2/L$.

1. (**2 points**) Draw the extensive form of this game.
2. (**2 points**) Show that a low-ability worker never pursues the program in a weak Perfect Bayesian Equilibrium (wPBE).
3. (**6 points**) Describe all pooling wPBE of the game (remember to specify the equilibrium fully). Does your answer depend on p_0?
4. (**4 points**) Describe all separating wPBE of the game.

Now, suppose that the firm also has an option to hire the worker after the first year of the MBA program. Formally, consider the following two-period game. First, nature determines the worker's type $\theta \in \{H, L\}$ with $\Pr(H) = p_0 \in (0, 1)$. After observing own type, the worker chooses whether to enter the two-year MBA program. The firm observes this choice, and at the end of period 1 decides whether to hire the worker or not. If not yet hired, the worker chooses whether to continue the program. The firm observes this choice, and at the end of period 2 decides whether to hire the worker or not. The firm's revenue from hiring a high-ability worker in the first period is $1 + r$ with $r > 0$, while revenue from doing so in the second period is 1. So, the firm gains from acting early. The cost of education is $1/\theta$ per period.

Assume that $1/L < w$ in addition to the assumptions on the parameters made previously.

5. (**6 points**) Does this two-period game have a separating pure-strategy equilibrium? That is, an equilibrium in which only the high type is hired (in period 1 or in period 2).